The zero-force hinge.

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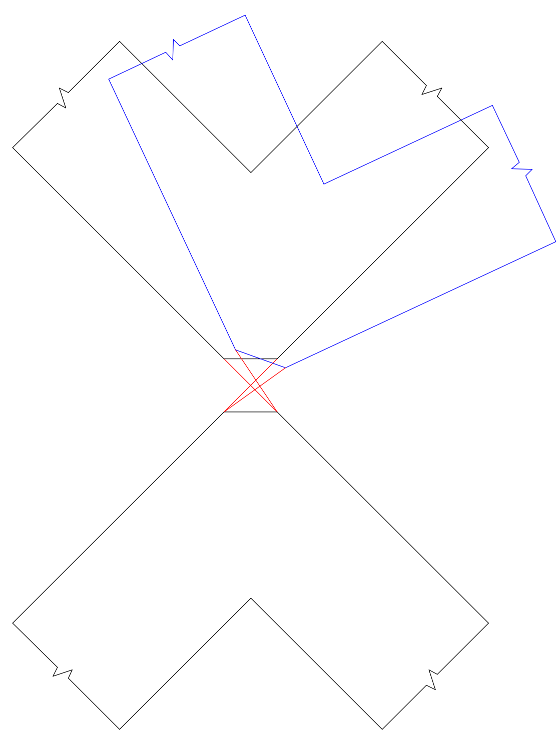
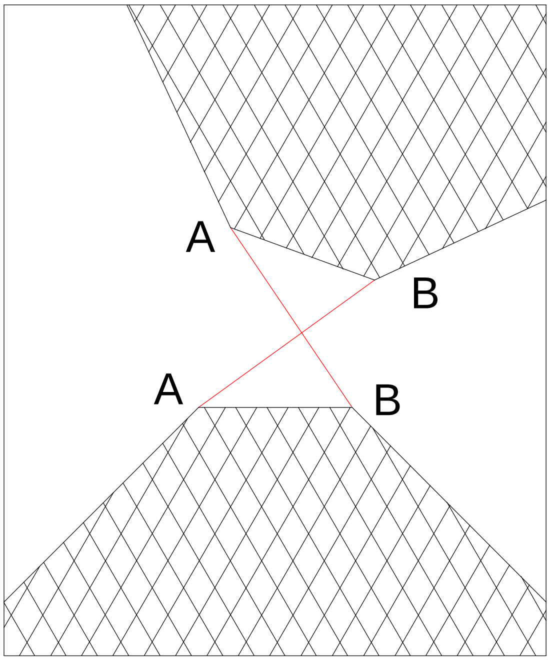
In attempting to analyze a leaf-spring vertical seismometer in order to predict the net restoring force on the system, it became evident that I would have to consider the effect of the forces arising in its hinge. Since accurately calculating the hinge forces appeared somewhat daunting I was left with the possibility of trying to redesign the hinge so as to reduce its restoring force to a value which could safely be neglected. The hinge design employed is the classical crossed-flexure configuration used with great success for many years, so I set out to see how far the design could be pushed to reduce the hinge forces. This led to the "zero-force" hinge design presented here. The configuration should more properly be called a zero-moment hinge, as it theoretically should have no restoring torque as it rotates. I should mention that this design has been used previously by George Harris in his vertical sensor, and considering its attractive properties, I would expect that it has previously found application in numerous other places.

In order to reduce the hinge forces, two parameters appeared to be most susceptible to modification. First, since the forces are inversely proportional to the third power of the flexure thickness, designing with the thinnest flexures possible became a goal. Also the forces are inversely proportional to the radius of curvature of the flexures, which implies that making them longer would be better. Those two goals meant that one could no longer count on the flexures to support any compressive forces. Such long, thin elements would buckle too easily, which says that the design would have to keep all the flexures in tension, under all conditions.

In studying the hinge configuration, I was interested in the general problem of the extent to which the design approximated a "perfect" hinge, rotating about a single point. I was also interested in the nature of the arc described by the end of a boom element mounted to such a hinge. This suggested that the analysis should be extended to include moderately large deflections of the hinge, although in the STM-8, for example, the hinge is actually required to flex less than ±0.2°.

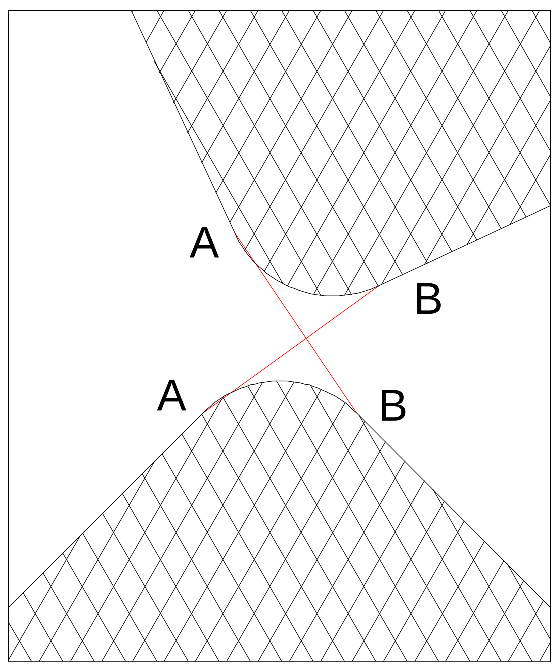


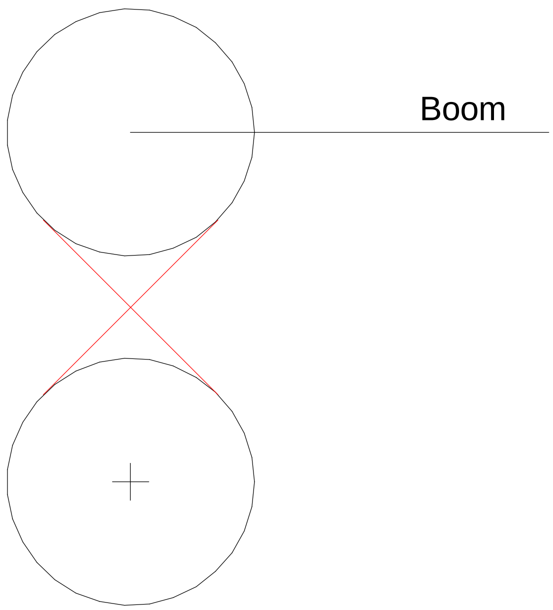
The hinge in that design consists of two 90° metal angles, placed so that their outside corner edges are nearly in contact, having four thin metal flexures bonded to their surfaces such that one pair crosses the other at right angles – the crossed-flexure hinge. It is desirable that the adjacent corners of the support angles be beveled-off sufficiently to create a small gap in which the flexures may work. This is shown, as viewed down the axis of the hinge, here shown rotated 45° from its original orientation in the STM-8.



An enlarged view shows the action of the flexures with the top support rotated by 10°. We can see that at the points marked A, the radius of curvature of the flexures is quite small, implying a larger than necessary bending force. At the points marked B, the flexures are tending to lift off their supporting surfaces, which makes calculating their forces and the hinge geometry somewhat uncertain. These effects may not be as pronounced when thicker flexures are employed, or when rotating through very small angles, but in the design we are seeking they might be troublesome.

One thought is to create a radius on the outside corner edges of the support angles. Now we see that at the points A, the curvature has become quite gentle, although at the points B, the flexures will still tend to lift off the supports.

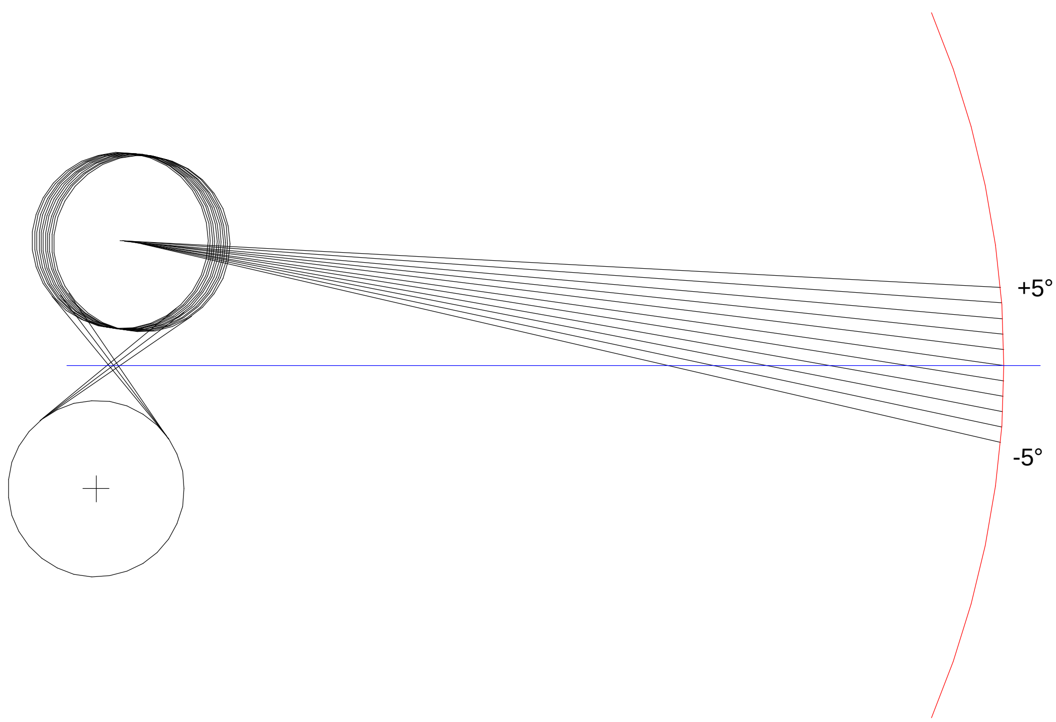




If we take the design to its logical conclusion, we can do away with the support angles completely, and replace them with two parallel cylindrical rods. The flexures wrap around the rods and are attached to the rods at points well removed from the operating gap. Though it is not necessary to the design, it is a good compromise to have the flexures cross at 90°. For that to be the case, assuming that both rods are of diameter d, they must be separated by a center-to-center distance of $\sqrt{2}$ d. The remaining analyses will be based on that configuration. The only assumption made is that the load forces are great enough and the flexures are thin enough that they will lie substantially tangent to the cylinders. For stiffer flexures, this might not always be the case.

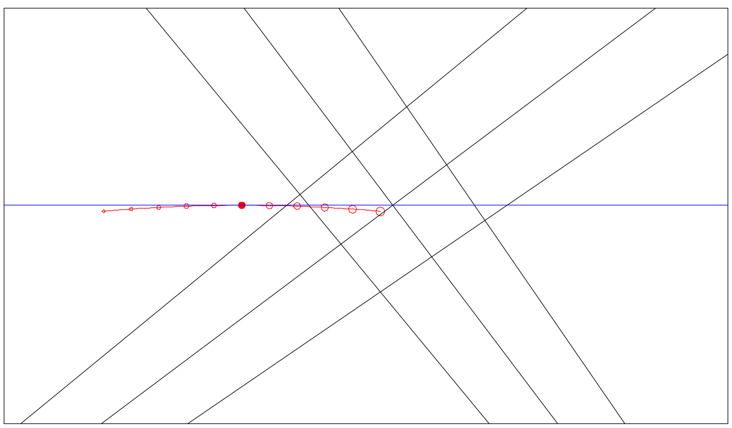
The rotation of the rods is quite interesting. A 1° deflection of the boom is the sum of two rotations. First the top cylinder and boom rotate 1/2° about its axis, plus there is an additional 1/2° rotation of the top cylinder and boom as a unit about the axis of the lower cylinder. Visualizing how this worked was eluding me until I borrowed two wheels from my grandsons' Tinker Toy set and added a figure 8 loop of string. After a few minutes of playing it was apparent that the hinge was moving as described above.

To see how well the hinge maintains its center, I used a CAD program to redraw the boom and top cylinder in 11 positions, each 1° apart, thus covering a range of 10°.



In this example, a particularly short boom was constructed, with its length only 5x the assumed cylinder diameters of 1”. If we had made the cylinders a more typical size, say between 0.25” and 1cm in diameter, we’re talking about an equivalent boom less than 2” long. Although this does demonstrate an extreme case, it is one however in which the motions of the parts are easy to see, and it should certainly provide a severe test of the geometry. The first item of interest is that the center of the arc made by the end of the boom appears to lie about midway between the cylinders. The “zero degree” position of the boom was chosen to be the position where the end of the boom was the farthest to the right, which occurs at the level of the 0° intersection of the flexures.

Since any three points (not in a straight line) will determine a unique circle, I used groups of three adjacent positions of the boom end to construct a set of circles. For example a circle was drawn through the +1°, +2° and +3° boom end positions. Its center was marked and called the center of rotation for +2°. Using this method, the centers of rotation were plotted for each boom position from –5° to +5°.



The smallest circle, on the left marks the center of rotation for +5°. The solid circle is 0° and the largest circle on the right is the center of rotation of the boom for -5°. The centers move about 0.009” for each degree of boom motion, covering a range of 0.092” over the ± 5° range. The pairs of intersecting lines are the flexures at the +5°, 0° and -5° positions. Over that range, the intersection of the flexures moves by 0.061”.

I was expecting to see a comparable error when I tried to fit a circle to the locus of boom end-points. However, the errors turned out to be much smaller than I’d expected. The reason is that each center point is associated with its own radius. The real effect of the moving center is to very slightly flatten the locus curve at the top and to very slightly deepen it near the bottom. The errors from that effect, are extremely small. for example, if a circle is drawn centered at the 0° center of rotation through the end of the boom at the 0° position, then at its +5° position, the boom extends 0.00043” past the circle and at the -5° position it falls short of the circle by 0.00043”, the other points on the locus all being closer. If the circle is drawn centered at the 0° intersection point of the flexures through the 0° boom end point, the locus is to the right of the circle everywhere else, but by no more than 0.00025” for any position.

With a longer boom, the fit is as good or better. It also appears that as the boom is made longer the 0° center of rotation approaches the 0° intersection point of the flexures.

I believe it is fair to conclude that this hinge design is capable of providing rotation about a center point with an accuracy which is quite adequate for any practical application.