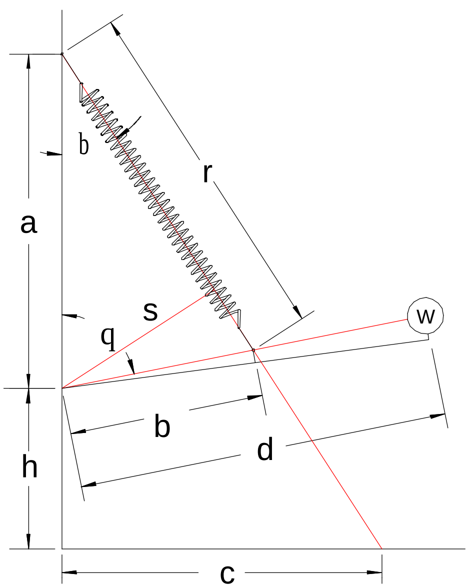
The geometry of a vertical seismograph

LaCoste demonstrated that a vertical seismograph could be constructed which had an essentially infinite natural period. His design consisted of a seismic mass at the end of a horizontal hinged beam, supported by a coiled extension spring. The force-length characteristics of the spring ("zero-length spring") in combination with the particular geometry which he used, created a system which in theory had zero net restoring force for any position of the beam. This provides the infinite natural period characteristic he was seeking.

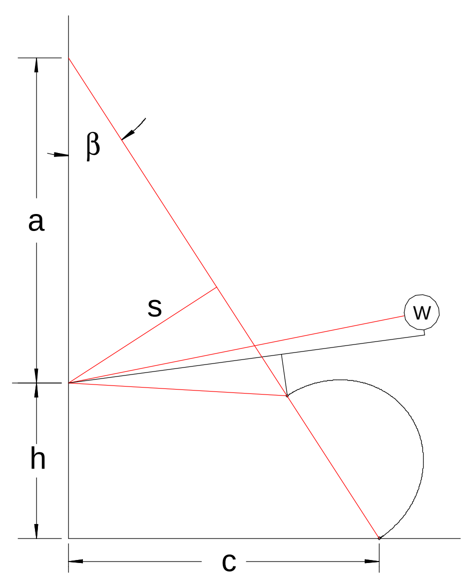
Another possible design for a vertical seismograph uses a bent leaf spring mounted under the beam as a compression spring to provide the restoring force. To what extent can this design achieve the same period-extending characteristics embodied in the LaCoste design?

First, let us examine the geometry of the LaCoste system and find out where it is similar to and where it differs from the leaf-spring design.



In the LaCoste design, a tension spring is attached to a vertical support at a distance **a** above the beam hinge. The mass **W** is the effective mass of the system, which consists mainly of the seismic mass and beam, and is located at the center of mass of that system. Thus the line connecting **W** to the hinge does not represent the supporting beam axis, but is rather the line drawn from the hinge through the center of mass. LaCoste's geometry requires that the spring be attached on that line. I am assuming here that the seismic mass is set above the centerline of the beam, and thus, the line between the center of mass and the hinge lies above the beam axis.

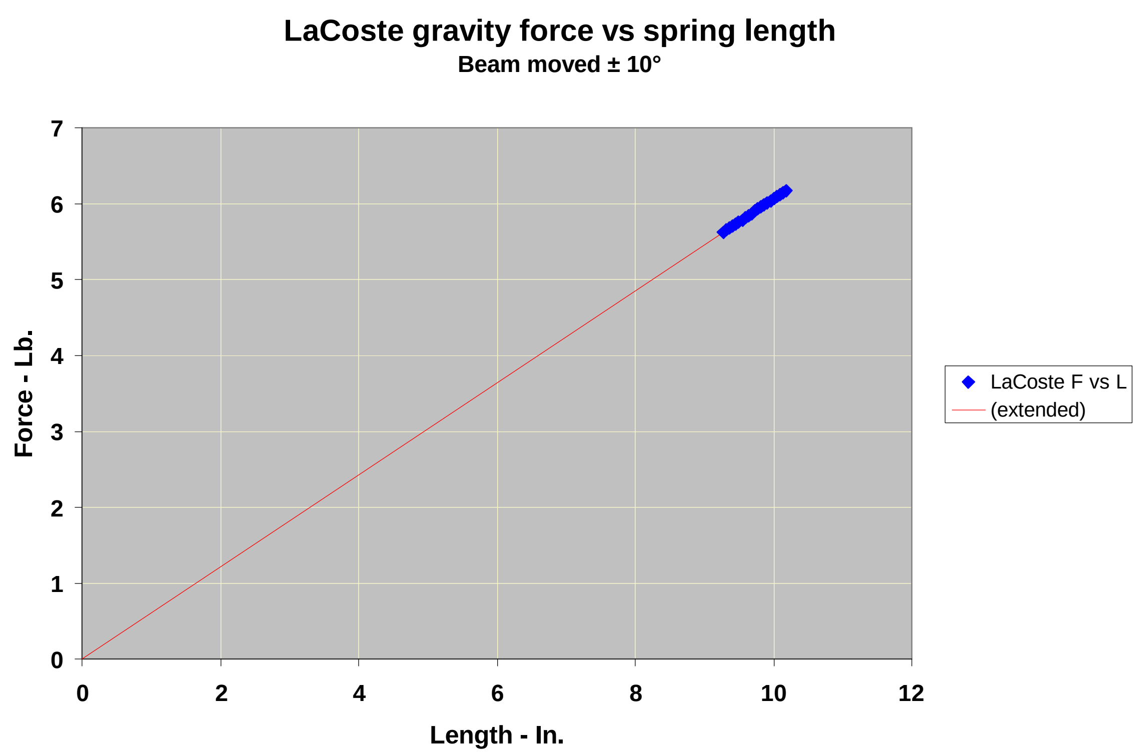
In the leaf-spring design, the spring is attached to the base, at a horizontal distance **c** from the vertical support. Succesful period-lengthening designs will probably have the line of action of the spring tilting back toward the vertical support, intersecting it at a distance, say **a** above the hinge. Also, the leaf-spring is attached to the beam at a point, fixed to the beam but assumed here to be below its centerline.



For any particular position of the beam, the effects of either a compression spring *pushing* up on the beam or of a tension spring *pulling* up on it can be made identical if they both exert the same force and act along the same line. Therefore, if the leaf spring attachments are positioned to lie along the line of action of a similar LaCoste configuration, the forces felt by the springs are identical. However, differences will appear once the beam moves up or down. In the LaCoste system the *upper* spring attachment is fixed and the line of action of its force will always pass through that point. In the leaf spring system, the *lower* attachment is fixed, and as the beam moves, the intersection of the line of action with the vertical support will move up and down. We should thus expect to see some differences in the behaviors of the two designs due to the differences in geometry.

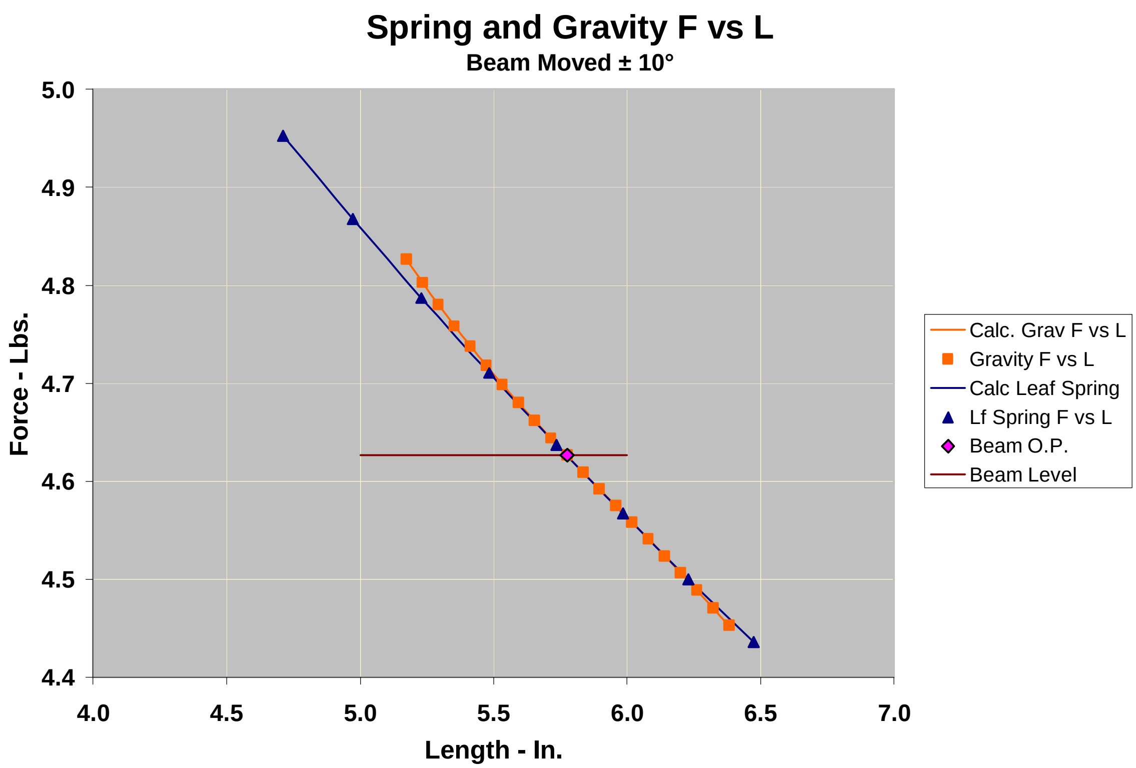
To compare the two systems, for each I computed the force required between the spring mounting points to balance the effect of gravity acting on the center of mass, for varying positions of the beam. Those variable forces were plotted against the corresponding distances between the spring mounting points. If we found and installed a spring which exactly matched the observed force-length characteristic, the spring's force would everywhere exactly balance the gravity force and we would have achieved an infinite period system.

With a randomly assembled LaCoste geometry, the force vs length plot is a beautiful straight line intersecting the origin. So a "zero-length" linear spring, designed to match the force line will exhibit infinite period with this geometry.



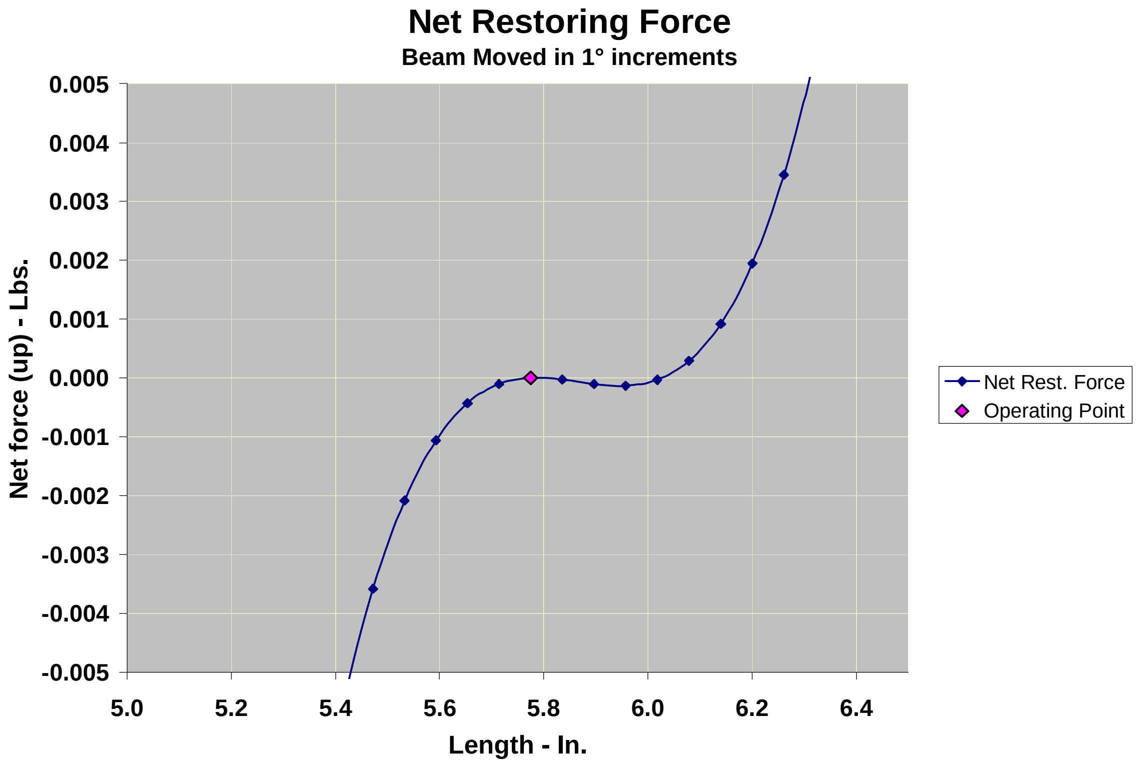
With the leaf-spring, the plot is a shallow curve, depending of course on the particular geometry and dimensions chosen.

But, do we need to achieve complete matching of the spring force with the gravity force for all positions of the beam as LaCoste does? Particularly with a feedback design, where the feedback acts to maintain a constant beam position, we don't. In that situation, we only need to be concerned with the forces when the beam is in the vicinity of its operating position. To have an infinite period, it is only necessary that the spring force curve be tangent to the gravity force curve at the operating point of the beam. The graph below shows the spring force vs length, plotted with the gravity force vs length for particular instrument dimensions which were selected to make the curves tangent when the beam is horizontal.



In such a design, if the feedback were removed, the beam would be unstable when positioned either above or below its operating point, and it would soon move to the limit stop. However with the feedback active, and with adequate DC loop gain, the beam should be easily held near the infinite-period position. The following graph of the difference between the gravity and spring forces as plotted previously, demonstrates the degree to which they can be made to balance. In this example, I assumed spring dimensions of 12" x 3.5" x 0.018" and starting from the assumed STM-8 dimensions, systematically adjusted them until the gravity and spring curves matched. Each plotted point represents one degree of beam motion, or about 0.24" of motion at the mass location.

Since the response characteristics of a feedback seismograph design do not depend much on the natural period of the spring-mass, it might be appropriate to ask, "why bother with all this"? I think the answer is that, by using a system having very little natural restoring force, there will be a number of major advantages when we attempt to implement the feedback system. The currents required in the feedback coil required to position the beam at DC are greatly reduced. In



addition, the loop gain at low frequencies, is much larger, which is needed if the electrical parameters are to solidly control the instrument response. As a practical matter, it would be desirable that the loop gain and dynamic range of the force feedback at 0 Hz (DC) be adequate to control the beam even near its extremes of position. Similarly, some provision would undoubtedly have to be made to get the beam centered and stabilized whenever the system was powered up.

[This is a first try at looking at this. I also plan to investigate procedures for determining real-world dimensions, and look at the effect of the geometry on the feedback loop design. Also references to be added.]

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