The temperature coefficient of a leaf-spring

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The leaf spring used in the STM-8 vertical seismograph has been observed to have a force which varies with temperature. This is an attempt to predict the amount of variation, considering only the material properties of the spring.

For simplicity, the variations will be expressed as fractional changes, in parts per million per degree C. (ppm/degC) One ppm = 0.0001%

Consider the spring at a specific amount of bend a degrees. For constant a , the spring force is proportional to $E\ T^3\ W$ Where E is the elastic modulus of the spring steel, and W and T are respectively the width and thickness of the spring.

For steel, E has a temperature coefficient of approximately –240 ppm/degC. The linear expansion coefficient for steel is +11 ppm/degC. Since these are small numbers, their combined effect will be approximately

–240 + 3 x 11 + 11 = -196 ppm/degC.

There is one small effect remaining. With a held constant the spring opening p is proportional to the unbent spring length L. So if L increases by 11 ppm/degC, so will p. So we must now return the gap opening to its original value before we're done, which adds a small additional force.

That force in ppm is given by df/f = -df/dp p/f dp/p
where

df/dp is the the slope of the spring curve at the point (p,f) and dp/p = 11 ppm.

For the STM-8, it was previously determined that, at a = 96 deg, f = 4.95 lb,
p = 4.71 in. and df/dp = -0.334 lb/in

Therefore our final term is

+0.334 x 4.71/4.95 x 11 = +3.5 ppm/degC (A lot of work for 3ppm!)

Adding that to the previous result we have the total spring force temperature coefficient of

-240 + (3 x 11) + 11 + 3 = -193 ppm/deg C

At the assumed STM-8 operating point the spring temperature coefficient is

4.95 lb x –193 ppm/degC = -9.6E-4 lb/degC

or in metric units

–434 mg/degC