A miniature wideband horizontal-component feedback seismometer

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Abstract A miniature, wideband, horizontal-component feedback seismometer has been developed and compared with conventional seismometers. The instrument employs an inverted pendulum of mass 0·04 kg supported with a natural period of 0·6 s and a Q factor of about 20. Displacements of the mass are sensed by a differential capacitive transducer and feedback is applied via an electromagnetic system to maintain the mass stationary with respect to its supports. The instrument has a response defined by feedback from DC to 100 Hz and has a Brownian noise level of 5×10^{-10} m s^{-2}. Overall dimensions, including an evacuated jacket, are 15 cm × 10 cm diameter making it suitable for borehole applications. The instrument has been operated for several weeks with conventional long-period seismometers and has been found to compare satisfactorily.

1 Introduction

The spectrum of earth motion of interest in seismology covers the range from about 0·01 to 10 Hz. A large peak known as microseismic noise occurs at about 0·15 Hz. It is due to effects on the earth's surface and seismometers have conventionally been designed to avoid it, short-period instruments operating over the range 1–10 Hz and long-period instruments from 0·1 to 0·01 Hz.

Seismometers measure the relative displacement x_r between a suitably supported mass and the instrument frame (assumed to follow the required ground motion). The equation of motion of the mass M when excited by a sinusoidal ground acceleration $\ddot{x}_i$ is

$$\ddot{x}_r + \frac{R}{M}\dot{x}_r + \frac{1}{MC}x_r = \ddot{x}_i \qquad (1)$$

where R represents the viscous damping resistance and C is the compliance of the supporting spring. The natural angular frequency is $\omega_0{}^2 = 1/MC$. It can be seen that for excitation frequencies $\omega < \omega_0$ the device behaves as an accelerometer with $x_r = \ddot{x}_i/\omega_0{}^2$, whereas for high excitation frequencies it becomes a displacement meter with $x_r = x_i$. Conventional seismometers have employed suspensions with natural periods of the order of 1–3 s for short-period instruments, and of the order of 10–30 s for long-period instruments in order to obtain the maximum response from the mechanical system. Various types of 'standard responses' have been adopted, often based on existing instruments, obtained by feeding the output to a suitable filter or by adjusting the period and damping of the instrument.

The fundamental limit to the detection of ground motion by a seismometer is set by the Brownian motion of the mass. It can be shown (Usher 1973) that the noise-equivalent acceleration $(\ddot{x}_i)_{ne}$ for a bandwidth Δf is given by

$$(\ddot{x}_i)^2_{ne} = \frac{4RkT\Delta f}{M^2} = \frac{4kT}{M}\frac{\omega_0}{Q} \qquad (2)$$

where kT is the equipartition energy and Q the quality factor of the suspension. A small mass may be used provided that the damping is low, though seismometers have conventionally employed large masses of several kilograms, usually nearly critically damped. A mass of 0·01 kg with a natural period of 1 s and a Q of 100 has $(\ddot{x}_i)_{ne} \approx 3 \times 10^{-10}$ m s^{-2} in a bandwidth of 1 Hz at room temperature, which is of the order required in practice (see later).

The application of negative force-feedback to a seismometer produces a number of advantages and is in fact necessary when a small mass is suspended with a high Q, in order to achieve a satisfactory transient response. Feedback may affect any of the three terms of the left-hand side of equation (1). The most useful form is negative displacement feedback, which tends to keep the mass fixed in position with respect to its supports and affects only the term in x_r, making the suspension appear more stiff and increasing the natural frequency. A general block diagram of such a seismometer system is

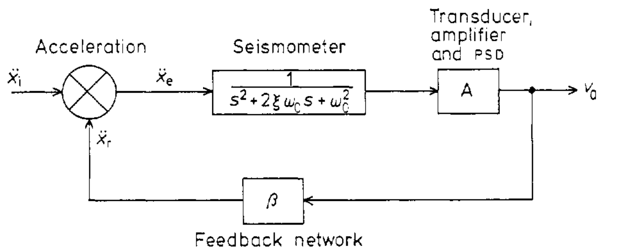

Figure 1 A force-feedback seismometer.

shown in figure 1, in which the transfer function of the seismometer itself is represented as

$$\frac{x_r}{\ddot{x}_i} = \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0{}^2}$$

where $s = j\omega$ is the Laplace operator and ξ is the damping ratio. The closed-loop transfer function is

$$\frac{v_0}{\ddot{x}_i} = \frac{A}{s^2 + 2\xi\omega_0 s + (\omega_0{}^2 + A\beta)}$$

where v_0 is the output voltage, A is the gain in the forward path and β is the transfer function of the feedback path, and becomes equal to $1/\beta$ when $A\beta$ is the dominant term.

If it is assumed that $A\beta$ is independent of frequency, the DC loop gain L is $(1/\omega_0{}^2)A\beta$ and the natural frequency is increased by a factor $L^{1/2}$, the damping being reduced by the same factor. The response is essentially flat (to acceleration) from DC to the new natural frequency and the transient response can be controlled by a compensation network in the feedback path.

A small mass of, say, 0·01 kg, suspended with a Q of 100 and employed in a feedback system of suitable loop gain (say 100), can thus provide a flat response and adequate detectivity over the whole range of interest in seismology. In addition, advantages over conventional open-loop instruments are obtained in linearity, dynamic range and calibration.

Mechanical design requirements are eased and the desired wideband response is simply determined by the feedback parameters. The signal-to-noise ratio is unaffected by feedback. The response is controlled by applying forces to the mass; this does not affect its Brownian motion, whereas adding damping to control the response in an open-loop system increases the dissipation and therefore increases the Brownian motion.

2 Review of developments in seismometry

The earliest seismometers employed very large masses, sometimes of several tonnes, in order to achieve a sufficient momentum of the mass to drive the recording equipment. Conventional seismometers typically have masses of 10 kg with periods of 10–30 s (Geotech S12 long-period seismometer) or 1 kg with periods of 1–3 s (Willmore Mk III short-period seismometer), and employ an electrodynamic transducer for sensing the position of the mass with near-critical damping.

A few miniature instruments have been developed, notably by Block and Moore (1970), Block and Dratler (1972) and by Jones (1967). Jones' instrument was a pendulum tiltmeter employing a mass of about 0·02 kg and a differential capacitance displacement transducer. It was operated open-loop with a natural period of about 0·3 s and had relatively high damping; while successful for recording very long-period (e.g. tidal) motions it was not designed for the range of interest in seismology and had a theoretical Brownian noise of about 5×10^{-9} m s^{-2}. The instruments developed by Block *et al* were miniature wideband seismometers in which a mass of 0·01–0·02 kg was supported by a quartz fibre in torsion with a natural period of about 1 s. A differential capacitance displacement transducer was used to sense the mass displacement. Although an electrostatic feedback system was proposed the instrument was operated open-loop with a fairly high Q (about 20) so that the transient response was not satisfactory and the instrument was not suitable for measurements at short periods without special filtering. The temperature coefficient for torsion of the quartz suspension resulted in a very high temperature stability requirement and the instrument was enclosed in two evacuated containers, its overall size being similar to that of conventional instruments.

A number of seismometers employing feedback have been described in the literature. Tucker (1958) applied force-feedback to a pendulum seismometer, whose natural period was about 1 s, by means of an inductive displacement transducer and electromagnetic force transducer. The loop gain used was only about 3 so that the response was not completely determined by feedback; the instrument was intended for the study of microseisms. Instruments described by de Bremaeker *et al* (1962) and by Sutton and Latham (1964) employed feedback at very long periods, but not in the seismic range. Block and Moore (1966) applied feedback to a conventional La Coste–Romberg instrument via a capacitive displacement transducer and electrostatic force transducer, but the forces available with this transducer are very small and the response in the range of interest was not controlled by feedback. As mentioned above, Dratler (1971) describes an electrostatic feedback system for a miniature seismometer, but the system was apparently not used in practice. Systems have been described by Willmore (1959), Russell *et al* (1968) and Kolez-nokov *et al* (1975, private communication) in which short-period Willmore seismometers have been modified by force-feedback, using the existing electrodynamic transducer for sensing the mass motion. In these instruments feedback modifies the response and produces advantages in linearity and calibration, but the detectivity is the same as for an open-loop, short-period instrument. Melton (1976) has discussed force-feedback seismometers but the instruments described employ relatively large masses (about 0·4 kg).

It can thus be seen that a miniature wideband feedback instrument has not previously been developed. The miniature open-loop instruments described above either are long-period devices with inadequate detectivity in the seismic range or have large overall size. The feedback instruments described in the literature mostly have low loop gain in the range of interest or are not miniature devices. We have been unable to find a reference reporting the use of substantial feedback over the seismic range accurately defining the instrument response and controlling the transient behaviour, with its attendant advantages of linearity, dynamic range and calibration.

3 Design philosophy of miniature wideband seismometers

The signal levels required to be detected can be deduced from the spectrum of background seismic noise. This has been investigated by many workers, notably Brune and Oliver (1959), Fix (1972) and Savino *et al* (1972). Figure 2 is based on the work of Fix, and shows acceleration power densities at a very quiet site (Queen Creek). The minimum observed acceleration power occurs at a period of 30–50 s and has a value of 10^{-19} (m s^{-2})^2 Hz^{-1}.

To detect a signal of this magnitude, the instrumental noise, determined by Brownian motion of the mass and transducer/amplifier noise, must be sufficiently small. The Brownian motion of the mass sets the fundamental limit to detection of signals, and the required mass/damping can be deduced from equation (2). A mass of 0·04 kg with a Q factor of 50 has a Brownian noise-equivalent acceleration very close to this value, and is shown as a horizontal line in figure 2 together with the Brownian noise levels for other combinations of mass and damping.

The transducer/amplifier noise should be designed to be less than the Brownian noise, in order not to degrade the detection limit. A suitable transducer is the differential capacitive type, which is essentially noiseless. The electronic noise is largely determined by the following amplifier and can be represented by a series noise-equivalent generator R_{nv}. The noise-equivalent acceleration can be shown to be

$$(\ddot{x}_{\mathrm{i}})_{\mathrm{ne}}=\frac{(s^2+2\xi\omega_0 s+\omega_0{}^2)}{r}(4\,R_{\mathrm{nv}}kT\Delta f)^{1/2} \qquad (3)$$

$$=\frac{\omega_0{}^2}{r}(4\,R_{\mathrm{nv}}kT\Delta f)^{1/2}\ \text{for}\ \omega<\omega_0$$

where r is the responsivity of the transducer. The natural period T_0 $(=2\pi/\omega_0)$ and responsivity r must be chosen such that $(\ddot{x}_{\mathrm{i}})^2_{\mathrm{ne}}<10^{-19}$(m s^{-2})^2 Hz^{-1} with the practical value of R_{nv}.

After the required minimum signal level has been chosen and the Brownian noise has been made of the same order as, and the electronic noise made less than this level, it is only necessary to make the noise from other sources (such as the phase-sensitive defector (PSD), filters, etc) negligible. This can be done by ensuring that there is sufficient gain prior to each element.

Finally, to achieve the desired wideband response of defined

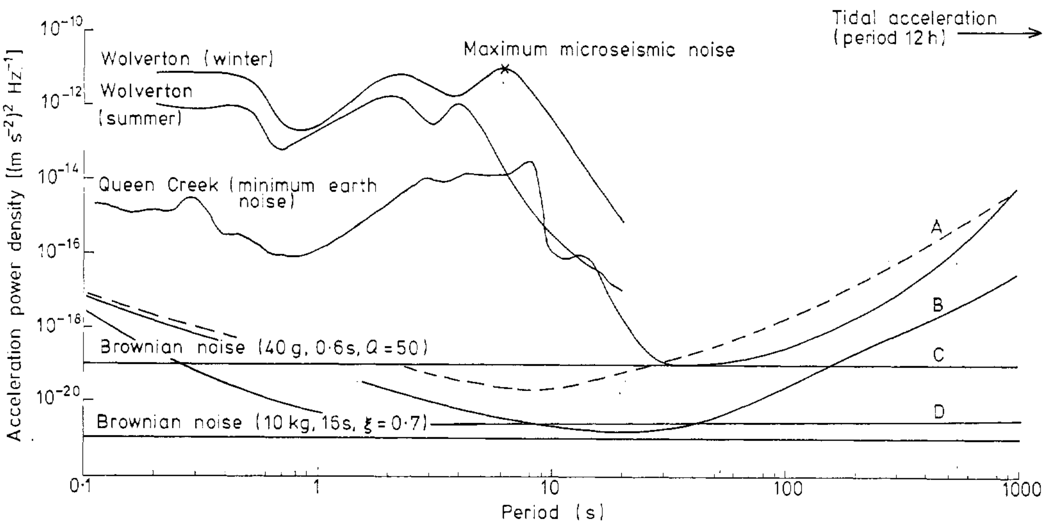

Figure 2 Spectrum of earth noise (after Fix). Acceleration power densities for Queen Creek (USA) and Wolverton (UK) are compared with the noise-equivalent accelerations of a standard seismometer (curves A and B) and with the Brownian and electronic noise levels of a miniature instrument (curves C and D). A, 10 kg mass with standard amplifier; B, 10 kg mass with special amplifier; C, Brownian noise of feedback seismometer; D, electronic noise of feedback seismometer.

magnitude and controlled damping, negative force-feedback is applied to maintain the mass position fixed with respect to the instrument frame.

4 Description of the instrument

4.1 *Mechanical system*

The basic requirements of the suspension system of the instrument are that the pendulum be constrained to move with a single degree of freedom and with a suitable fundamental period and Q factor. All other modes must be far removed from the frequencies of interest, and above the feedback loop cut-off frequency, and the long-term stability must be satisfactory.

It can be seen from equation (3) that the natural period T_0 must be large enough to make the amplifier noise negligible. A natural period of about 1 s was achieved by using an inverted pendulum supported by simple spring strips. These produce a smaller restoring torque than cross-spring pivots and can easily be made from a material of low temperature coefficient such as Ni-span D. Our earlier investigations of period-lengthening devices showed that these become very critical when used with small masses, and the arrangement used is much more compact and robust.

The transducer must have high responsivity and low noise, combined with adequate long-term stability. A differential capacitance transducer is excellent in these respects (Jones and Richards 1973), providing a higher responsivity than other available transducers such as linear variable differential transformers (LVDTs), and a high detectivity. The AC operation of such a transducer avoids the $1/f$ noise region, which is a major problem in the design of (DC) amplifiers for conventional seismometers.

The basic instrument is shown in figures 3 and 4, with the heater cylinder and outer jacket removed. The pendulum has a mass of about 0·04 kg and is supported by two spring strips (of Ni-span D) clamped at both ends. The main frame is machined from a single piece of brass and supports the outer plates of the differential capacitor, insulated from it by quartz spacers. The pendulum itself is the central plate of the differential capacitor, insulated by quartz spacers, the signal to the preamplifier being taken via the tag shown. The components in the main frame assembly are kinematically mounted, permitting easy dismantling and assembly and improving stability. A high-stability magnet is attached to the centre of the pendulum bob, the coils of the force-feedback transducer being attached to the outer plates of the capacitor; this arrangement was found to be necessary to avoid a high-frequency instability in the closed-loop response when the feedback force did not coincide with the centre of mass. The thermal enclosure fits over the main-frame unit, with heater windings on the outside, and supports two electronic boxes containing the preamplifier/excitation and thermal control electronics, as shown in figure 5. The outer jacket fits over an O-ring seal and can be evacuated via a tap. It is attached to an

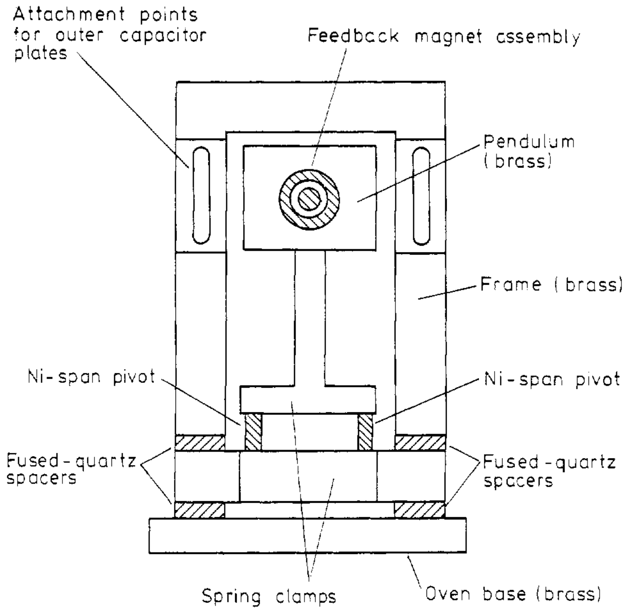

Figure 3 Diagram of seismometer with heater and cover removed.

Figure 4 Photograph of seismometer with heater and cover removed.

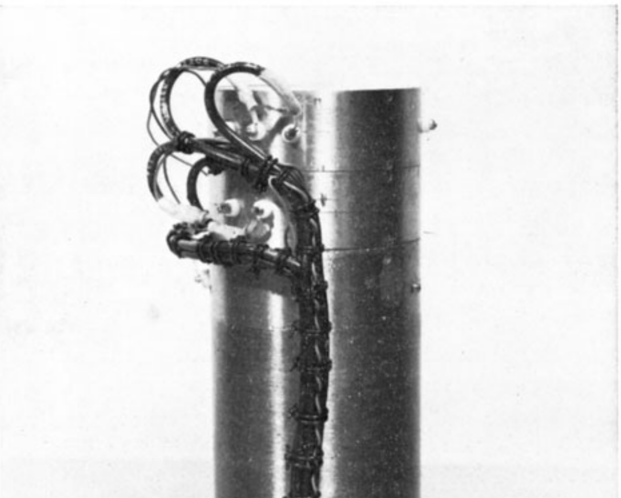

Figure 5 Photograph of seismometer showing heater and internal electronics.

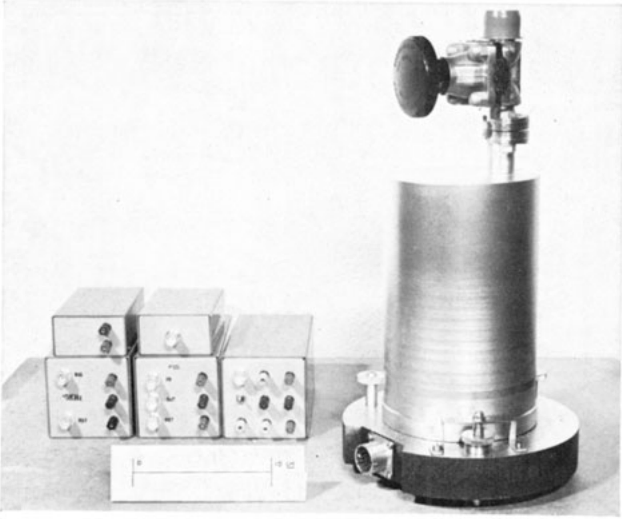

Figure 6 Photograph of complete instrument and electronics.

Invar base plate which contains a precision differential levelling arrangement and the socket for the electronics cable. The complete instrument and the associated electronics are shown in figure 6.

4.2 *Electronic system*

The essential requirement of the servo-system is that the response of the instrument be independent of the suspension and be defined only by the passive feedback elements over the whole range of operation from DC to 10 Hz. The loop gain should be about 100 over this range, and the response should preferably be flat to input acceleration or velocity and the damping near critical.

A block diagram of the complete system is shown in figure 7. The input ground acceleration $\ddot{x}_i$ produces a relative displacement x_r (between the mass and the frame) which is converted to a voltage v_r at the excitation frequency by the transducer and preamplifier and further amplified by the channel amplifier. This signal is demodulated by the PSD, and the feedback network drives the feedback coils to return the mass towards its original position.

The method outlined above (§1) can be implemented directly, but it is advantageous to introduce an integration into the loop to provide high gain at low frequencies. Mass displacements due to large tidal forces or thermal drift are thus reduced and the linearity of the transducer is improved. In addition, the design requirements of the PSD are eased, the necessary dynamic range is reduced and a greater $1/f$ noise level can be tolerated. If the time constant of the integration is made fairly large (e.g. 1 s) the behaviour of the system can be very similar to that of the simple proportional system, with a closed-loop resonance frequency of, say, 10 Hz. However, some difficulties were encountered with system stability in such an arrangement, due to poor recovery from overload and to stray mechanical resonances at high frequencies, and a modified arrangement was used in practice. The integration time constant was made 0·15 s producing a break in the response at 0·35 Hz; the response was then flat to acceleration at frequencies below 0·35 Hz and flat to velocity above this frequency. It is still defined by feedback, of course, and has the additional advantages that high-frequency ground motion does not produce an excessive output and that filtering to obtain the velocity response normally required is easier in some cases.

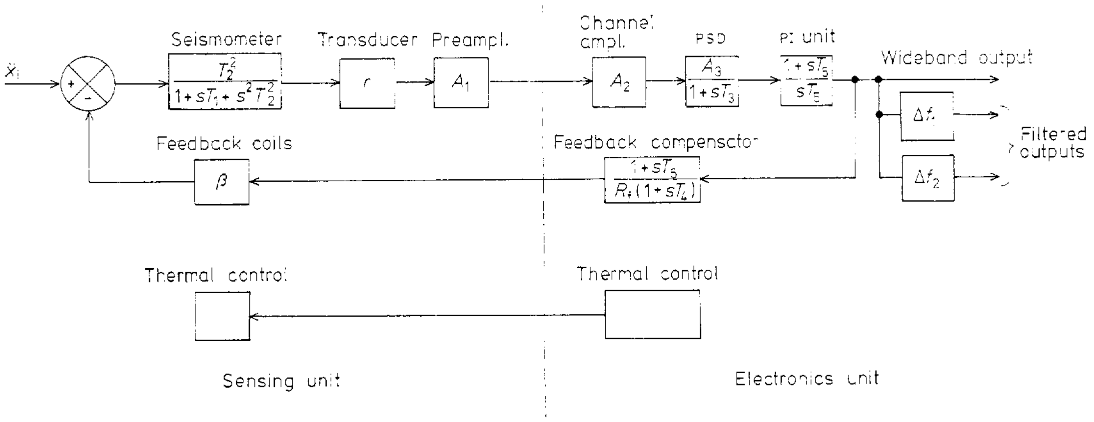

Figure 7 Block diagram of complete system. The input acceleration produces a relative displacement between sensing mass and instrument frame, which is detected by a displacement transducer, amplified and rectified, and fed back to maintain the mass stationary with respect to the frame.

4.2.1 *Capacitive transducer and preamplifier* The responsivity of a linear electrical displacement transducer is always of the form excitation voltage/range, and it is because the range can be made small that a high value of responsivity can be obtained. A responsivity of $10^4\ \mathrm{V\,m^{-1}}$ was obtained by applying 3 V RMS to the outer plates with a plate separation of 0·3 mm. A smaller separation leads to difficulty in levelling and increases the electrostatic forces on the mass, which must be kept small and constant. With the values used the electrostatic force has a value of 2×10^{-8} N when the centre plate is displaced by 10 μm (which just overloads the PSD on open loop).

The transformer arms and differential capacitor form a Blumlein bridge, of which the equivalent circuit is shown in figure 8. The series capacitance $C\approx20$ pF and the stray

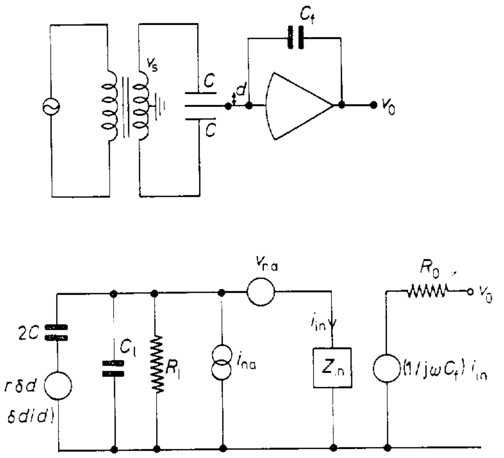

Figure 8 The Blumlein bridge and its equivalent circuit.

capacitance $C_1\approx10$ pF. It can be seen that stray capacitances do not affect the balance point of the bridge. The resistor R_1 is mainly determined by the biasing resistors in the preamplifier, and has a value of about 10 MΩ.

The charge amplifier configuration has the advantages that the signal level is not affected by changes in stray capacitance and that the output is independent of frequency over a wide range. The effective series noise-equivalent resistance referred to the signal source can be shown to be

$$R_{\mathrm{n}}=R_{\mathrm{nv}}\left[1+\left(\frac{C_1+C}{C}\right)+\frac{1}{\omega CR_1}\right]^2+\frac{1}{\omega^2C^2}\left(\frac{1}{R_{\mathrm{ni}}}+\frac{1}{R_1}\right)$$

where R_{nv} and R_{ni} are the noise-equivalent resistances of the input device, which was a low-noise FET. Using the values given above and assuming that $R_{\mathrm{nv}}\approx500\Omega$, $R_{\mathrm{ni}}\approx10\ \mathrm{M\Omega}$, we find that the optimum noise performance occurs at a frequency of 100 kHz. The noise referred to the input is 5 nV RMS in a bandwidth of 1 Hz, corresponding to $R_{\mathrm{n}}=1{\cdot}5\ \mathrm{k\Omega}$. This low value was possible because of the low value of C_1, due to the quartz spacers, and the high values of R_1 and R_{ni}. The value was confirmed experimentally. The voltage gain was 70.

The preamplifier was mounted close to the transducer to reduce cable capacitance. The channel amplifier A_2 is of conventional design, and was placed in the main electronics block, remote from the seismometer, together with the PSD feedback electronics, excitation oscillator and thermal control power amplifier.

4.2.2 *Drive oscillators* The stability of the excitation oscillator is important because of the very small displacements and forces involved. The least detectable acceleration of 3×10^{-10} $\mathrm{m\,s^{-2}}$ is equivalent to a force on the mass of about 10^{-11} N, and a high amplitude stability is therefore required to maintain the electrostatic forces sensibly constant. A Wien bridge oscillator was employed, in which the amplitude was controlled by a light-dependent resistor in the loop, the light level being derived by comparing the rectified output with a high-stability voltage reference. The amplitude stability was about 10 μV RMS over periods of up to 1 min and was found to be satisfactory.

4.2.3 *Phase-sensitive detector* The large dynamic range and low frequency of seismic signals make considerable demands on the properties of the PSD in terms of linearity, rejection of unwanted signals and noise level. The design employed was a complementary current-switching type similar to those developed by Faulkner and Grimbleby (1967), Danby (1968) and Grimbleby and Harding (1971). In order to achieve the necessary dynamic range and a good temperature coefficient the circuit employs a current mirror to produce a single-ended output (Buckner 1975). The output temperature coefficient was 0·001% (equivalent to 1 ppm°C^{-1}) and the noise about 10 μV RMS in a frequency band from 0·01 to 1 Hz.

4.2.4 *Proportional plus integral control unit* (PI) The PI unit comprises a high-quality operational amplifier providing an integration at frequencies up to 0·35 Hz and with a high-frequency gain of unity. Its noise and drift characteristics are important, since the wideband output voltage is taken directly from it, but are eased by the prior gain.

4.2.5 *Feedback compensator and feedback coils* The compensator is a passive, phase-lead circuit providing velocity-dependent feedback to stabilise the response. At medium frequencies the response is determined by a series resistor R_f in parallel with a capacitor C_f.

The feedback actuator comprises a high-stability magnet attached to the mass and small coils wound on formers attached to the centre of the outer capacitor plates. The acceleration coefficient was determined by applying a known tilt via a calibrated tilt table and measuring the current needed to reduce the output to zero (on open loop). The value of $1{\cdot}41\ \mathrm{m\,s^{-2}\,A^{-1}}$ was constant over the full ± 10 V range of the output and no departure from linearity could be measured. Since mass movement on closed loop is negligible, linearity is required only between force and current. The maximum acceleration produced before overload at 10 V with $R_f = 15\ \mathrm{k\Omega}$ is approximately $10^{-3}\ \mathrm{m\,s^{-2}}$.

The stability of the magnet was very satisfactory, and no significant drift attributable to this was detected in six months' continuous operation of the instrument. The evacuated jacket used for environmental control was of mild steel and provided very effective magnetic shielding.

4.2.6 *Environmental control* The basic pendulum is enclosed in a temperature-controlled oven, which also encloses the preamplifier, transducer excitation transformer and temperature-sensing circuit; the whole assembly is enclosed in an evacuated case at a pressure of about 10 Pa. Evacuation was necessary to eliminate atmospheric and other similar effects, and to achieve a suitably high Q factor for the suspension.

Although the instrument was designed to be mechanically as symmetrical as possible to reduce the effects of thermal gradients, its measured temperature coefficient was $10^{-6}\ \mathrm{g\,°C^{-1}}$. A stability of 10^{-4} °C would be required to achieve a DC resolution of 10^{-10} g (the requirement in Block and Dratler's instrument (1972) was a stability of 10^{-6} °C). The thermal control system employs a type YSI 4001 thermistor in an AC bridge operated at 1 kHz, and a PSD similar to that described above feeds a modulator and AC power amplifier. The heater coil consists of bifilar-wound copper wire on a groove in the outside of the brass heater cylinder; AC power was necessary to avoid magnetic effects. The parameters of the control system were adjusted experimentally to give a satisfactory response to a step rise in temperature.

4.2.7 *Output filters* The wideband output from the instrument is proportional to ground acceleration from DC to 0·35 Hz and proportional to ground velocity from 0·35 to 100 Hz. Outputs from seismometers are usually required in the form of various agreed 'standard responses' and the wideband output was therefore fed to a filter unit outside the loop to achieve whatever overall response was required. The system design ensured that noise and drift due to the filters were negligible.

5 System response, calibration and noise level

With reference to figure 7, the acceleration $\ddot{x}_r$ produced by the feedback coil on open loop in response to an input acceleration $\ddot{x}_i$ is given by

$$\frac{\ddot{x}_r}{\ddot{x}_i} = \frac{K(1+sT_5)(1+sT_6)}{s(1+sT_1+s^2T^2{}_2)(1+sT_3)(1+sT_4)}$$

where

$$K = \frac{A_1A_2A_3r\beta}{\omega_0{}^2T_5R_f}, \qquad T_1 = \frac{2\xi}{\omega_0}, \qquad T_2{}^2 = \frac{1}{\omega_0{}^2};$$

r, A_1, A_2 and A_3 are the gains of the transducer, preamplifier, channel amplifier and PSD respectively, $\omega_0{}^2$ is the natural angular frequency on open loop, β is the feedback force constant and R_f the series feedback resistor. T_1 and T_2 are time constants associated with the suspension, T_5 is the integration time constant, T_6 and T_4 refer to the phase-lead circuit and T_3 refers to the PSD. Time constants associated with the transducer, amplifier and feedback coil are less than 1 ms and are omitted. β is independent of frequency over the seismic range.

The open-loop and closed-loop responses are shown in figure 9, with $A_1 = 70$, $A_2 = 5$, $A_3 = 5$, $r = 10^4\ \mathrm{V\,m^{-1}}$, $\beta = 1{\cdot}4\ \mathrm{m\,s^{-2}\,A^{-1}}$, $\omega_0 = 10\ \mathrm{rad\,s^{-1}}$, $R_f = 15\ \mathrm{k\Omega}$, $C_f = 30\ \mu\mathrm{F}$, $T_1 = 0{\cdot}025$ s, $T_2 = 0{\cdot}1$ s, $T_3 = 0{\cdot}001$ s, $T_4 = 0{\cdot}001$ s, $T_5 = 0{\cdot}15$ s, $T_6 = 0{\cdot}45$ s.

The open-loop gain has a value of over 40 dB over most of the range of interest (DC to 10 Hz) so that the response is accurately defined by feedback. The closed-loop response is flat to acceleration (of value $R_f/\beta = 10^4\ \mathrm{V\,(m\,s^{-2})^{-1}}$) from DC to 0·35 Hz, and is flat to velocity (of value $1/\beta C_6 = 2{\cdot}4\times10^5\ \mathrm{V\,(m\,s^{-1})^{-1}}$) from 0·35 Hz to the unity open-loop-gain frequency of 100 Hz.

The dynamic response of the instrument was measured by applying square or sinusoidal waveforms to one of the feedback coils, equivalent to an input acceleration, and the curve obtained agreed very closely with that of figure 9. The step response had a rise time of 0·1 s and an overshoot of about 20%, corresponding to $\xi = 0{\cdot}7$. The DC responsivity was measured using a calibrated tilt-table and agreed with the value above.

The theoretical noise-equivalent acceleration can be deduced from equations (2) and (3). For a mass of 0·04 kg and a Q factor of 50, equation (2) gives $(\ddot{x}_i)_{ne} \approx 3\times10^{-10}\ \mathrm{m\,s^{-2}\,Hz^{-1}}$

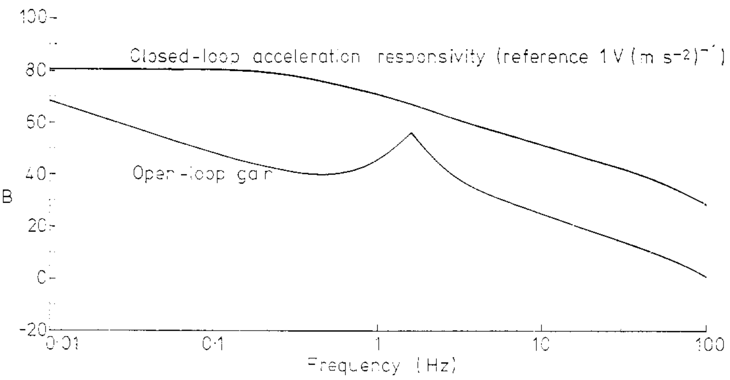

Figure 9 Open- and closed-loop responses of the feedback seismometer. The open-loop gain is greater than 40 dB over most of the seismic range (0·01 to 10 Hz). The closed-loop response is flat to acceleration from DC to 0·35 Hz (responsivity $10^4\ \mathrm{V\,(m\,s^{-2})^{-1}}$) and flat to velocity from 0·35 to 100 Hz (responsivity $2{\cdot}4\times10^5\ \mathrm{V\,(m\,s^{-1})^{-1}}$).

for the Brownian contribution. The transducer/amplifier contribution with $R_n = 1{\cdot}5$ kΩ and a natural period of 0·6 s gives $(\ddot{x}_i)_{ne} = 5\times10^{-11}$ m s^{-2} Hz^{-1} for $\omega < \omega_0$, which can be ignored. The other electronic sources of noise (PSD, filters, etc) have sufficient prior gain to make their contributions negligible. The theoretical noise-equivalent acceleration of the instrument is plotted in figure 2 (lines C and D).

The dynamic range of a wideband seismometer is particularly important, since seismic signals have a very large dynamic range. According to figure 2, the peak microseismic noise occurs at a period of about 8 s and has an acceleration amplitude of about 3×10^{-6} m s^{-2}. This is an average value (Wolverton in summer) and short-term values may be an order of magnitude greater. The 12 h tidal component has a similar value. The minimum earth noise (and the designed detection limit of the instrument) has an acceleration amplitude of 3×10^{-10} m s^{-2}. A range of about 80 dB is therefore required, excluding any additional range to accommodate excessive microseismic activity or seismic events. The maximum feedback acceleration is 10^{-3} m s^{-2}, and the effective dynamic range of the instrument is therefore about 130 dB, allowing about 50 dB for events. In practice, however, the recording systems employed have a range of only 60–80 dB, so that several systems of different gain, bandwidth, etc, would be required to record the output of the instrument satisfactorily.

6 Testing and operation

The instrument has been operated over a period of several months in the AWRE vault at Wolverton, and compared with a Geotech S12 horizontal-component long-period seismometer. This instrument has a mass of 10 kg suspended with near-critical damping with a natural period of about 20 s; it employs an electrodynamic transducer and low-noise amplifier (type 610) and has overall dimensions of 70 cm × 30 cm × 30 cm.

The temperature variations in the vault are about 3°C per day but it was found unnecessary to use the thermal control system. This was switched off while investigating the source of a lack of coherence at long periods, and was not used again although the source was located elsewhere. The feedback instrument did not show an appreciable long-term drift and the results suggest that a period of six months without adjustment is feasible. The drifts observed were, however, about an order of magnitude greater than the earth tides, which we did not attempt to observe. The Q factor of the instrument was about 50 immediately after evacuation, and slowly fell to a value of about 15 after about four weeks.

The comparisons were carried out using various standard responses, obtained by filtering the wideband output of the feedback instrument. The responses used were a short-period response derived from a long-period instrument (LPSP), a broadband response (BB), and a long-period narrowband response (LPNB). The outputs were telemetered to Blacknest and recorded together on standard 'helicorders' at suitable magnifications.

Good coherence was obtained with the LPSP and BB responses, as expected because of the relatively high seismic noise in these bands. There was some excess high-frequency noise in the feedback instrument, probably due to its wider bandwidth. The outputs from the two instruments initially showed rather low coherence when using the LPNB response even though their responses had been closely matched. This was thought to be due to thermal fluctuations in the base plate of the feedback instrument. The improved Invar base plate described above was constructed and led to better coherence. Figure 10 shows an event recorded at low magnification and figure 11 shows typical long-period noise at high magnifica-

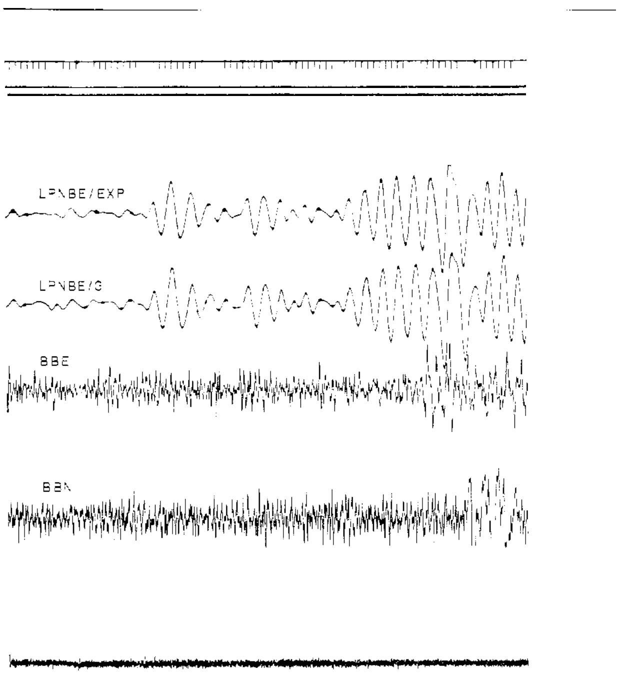

Figure 10 Comparison of seismometers (small event). A small event is shown at a magnification of 8200 at period 20 s using a long-period narrow-band (LPNB) response. LPNBE/EXP is from the feedback seismometer in the east–west orientation and LPNBE/G from the standard Geotech instrument. Two broadband records are also shown. (Event recorded at 03.30.00, 1 October 1975; scale, 8 s/division.)

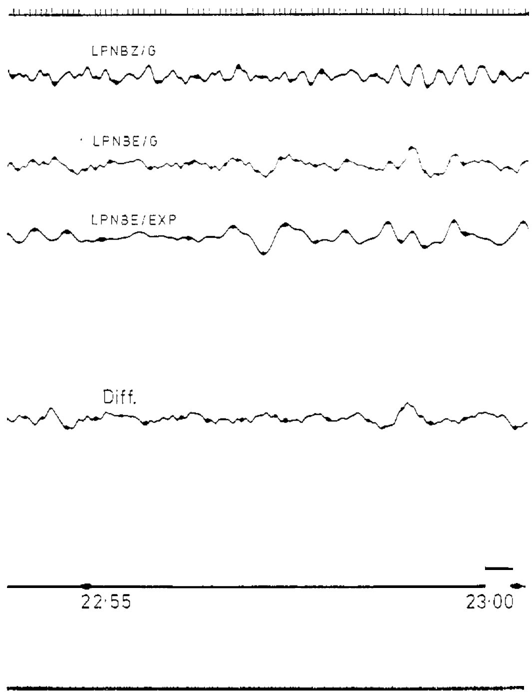

Figure 11 Comparison of seismometers (long-period noise). Long-period noise is shown at a magnification of 200 000 at 20 s using the LPNB response. LPNBZ/G is from a vertical-component Geotech instrument, and LPNBE/G and LPNBE/EXP are from the horizontal-component Geotech and feedback instruments. (Recorded at 22.55.00, 30 September 1975; scale, 4 s/division.)

tion. In the latter figure both recorded traces correspond to about 10 nm RMS at a period of 20 s (acceleration 10^{-9} m s^{-2}) in the LPNB bandwidth of about 0·04 Hz. The theoretical Brownian noise of the feedback instrument (with $Q=15$) in this bandwidth is 10^{-10} m s^{-2} and the theoretical electronic noise in the Geotech instrument has a similar value. Unfortunately it was not possible to compare two feedback instruments with one another (since only one was constructed) and two similar Geotech instruments were not available at the time.

The difference between the Geotech and feedback seismometer traces is mostly small with an RMS value of about 2 nm, corresponding to an RMS acceleration of 2×10^{-10} m s^{-2} at 20 s, which is close to the value expected theoretically from the Brownian noise value. However, occasional long-period fluctuations occur, with a magnitude of about 10 nm, probably due to mass movements caused by thermal effects or pressure changes. It is difficult to decide from the records whether the effect, which is very common in long-period instruments, occurs only in the feedback seismometer or the conventional seismometer or both, but it is more likely to be mainly due to the former.

The main practical problems in miniature instruments arise from long-term creep, thermal and pressure effects in the mechanical system, and air movements. As explained above, long-term creep did not prove to be a serious problem, and although thermal variations were observed their period was outside the seismic range of interest. (To observe longer-period seismic signals the thermal control system would have been required.) Pressure effects can be very serious, since the mass movements to be detected in a miniature instrument are smaller than in a conventional instrument in the ratio of the squares of the periods ($(0{\cdot}6)^2:(20)^2$). Similarly, unwanted forces due to air currents are equivalent to accelerations in the ratio of the masses (0·04 kg:10 kg). An evacuated pressure chamber is thus essential, though evacuation is required in any case to obtain a high Q factor. It is likely that the long-period fluctuations observed were due to thermal effects in the base plate of the instrument (the effect was much reduced by using the Invar base plate) and to pressure fluctuations leading to distortions of the base plate or instrument frame. In Block and Dratler's instrument the temperature was controlled to 10^{-6} °C and the pendulum was enclosed in a highly evacuated container and by an evacuated outer cover. However, it appears that the symmetrical design of the present instrument and the use of a rigid evacuated jacket and special base plate have enabled the above problems to be reduced to tolerable levels.

The comparison has successfully demonstrated that a miniature wideband feedback instrument can compare closely with a conventional open-loop instrument of much greater size. The increased electronic complexity is more than compensated by the reduced mechanical complexity, size and weight, and by the ease of operation and calibration. The cost, on a production basis, would be considerably less.

7 Conclusions

It has been demonstrated that it is feasible to build a truly miniature wideband seismometer with an output linearly proportional to ground motion and well defined by substantial negative feedback over the whole of the seismic range, and with a noise level comparable with much larger conventional instruments. The development depends strongly for its success on the use of a capacitive displacement transducer of high responsivity and low noise, and on the application of negative feedback to maintain the mass fixed with respect to its supports giving the attendant advantages of controlled wideband response, linearity and dynamic range.

A difficulty in designing a miniature seismometer is that, since a long natural period cannot easily be used, the displacements to be detected are very small and may become comparable with unwanted movements due to creep, air currents, etc. These problems were overcome by a carefully designed suspension system in which effects of temperature gradients were minimised. Although an evacuated cover and a thermal control system were used, the requirements of these systems were not stringent and the overall size is adequate for borehole applications.

It is, of course, easier to design a horizontal-component instrument than a vertical-component instrument, since the steady acceleration of gravity must be cancelled in the latter. However, the same principles can still be applied and an apparently satisfactory vertical-component instrument of similar overall dimensions is at present being tested.

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The authors wish to thank Professor Fellgett of the Cybernetics Department, University of Reading, who initiated this research and who first suggested that a combination of low damping and feedback could give a high detectivity even with low mass. The work was supported initially by a research grant from NERC and later by a research contract from AWRE. This contract arose through the initiative of Dr Thirlaway of the AWRE Seismological Centre at Blacknest, and the authors are much indebted to him for his continued help and encouragement and for the provision of testing facilities at Blacknest and Wolverton.

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