III. The STM-8 Loop

This contains the loop functions for a velocity input. The interaction between, forward gain, feedback function, loop gain and transfer function are well displayed. The asymptote functions give a simple way to estimate loop performance. More detail to mathematics. Some general design considerations added.

This is an analysis of the feedback loop of the STM-8 vertical seismometer. The results agree closely with the MathCad analysis done by S-T Morrissey. My goal here is to clarify, mostly in my own mind, the structure of the feedback system, and to that end I want to block-diagram the system in the form that I am used to using. In particular, I would like to derive a view of the loop which is aimed as much toward the synthesis process as it is toward analysis. For more information on feedback analysis, see the feedback tutorial file “feedback.pdf”.

The analysis is done with respect to an inertial frame of reference fixed to the earth’s surface. The input signal is the force on the mass due to ground motion. The feedback is the force from the Feedback Coil. The mass itself is the summing point.

There are three branches in the feedback path. The two outputs on the right, in blue, are independent of the loop and can be analyzed separately; that is, they are independent if there are no significant loading effects, which I think is the case.

The next step will be to insert the appropriate transfer functions for each block. The spring Mass will be a quadratic, the others will be quite simple functions.

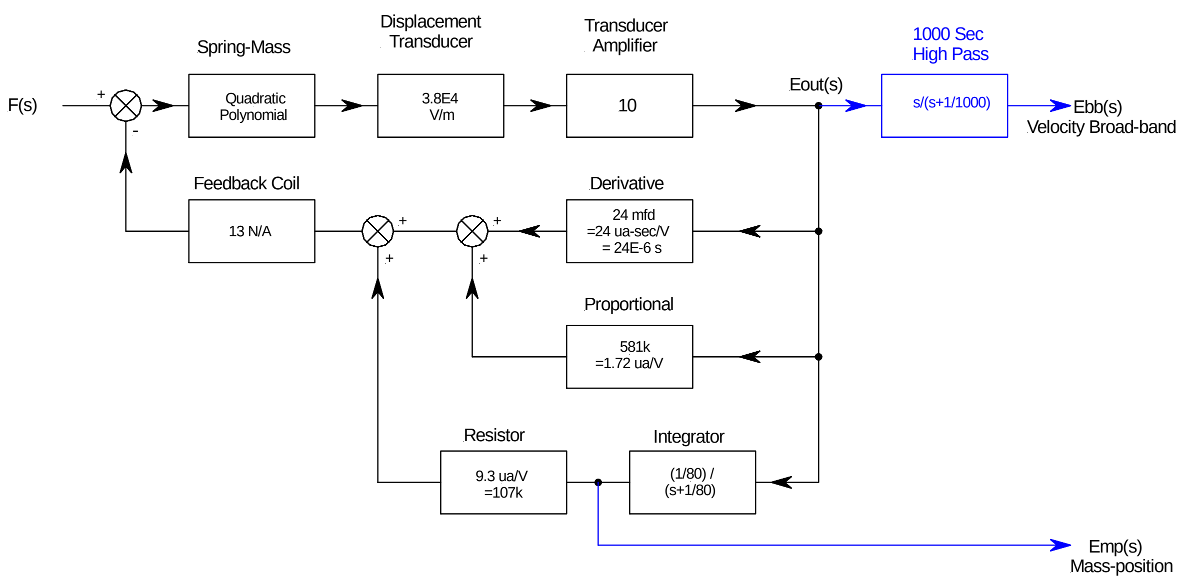

Figure 1 – Original STM-8 configuration

To analyze this loop we want to reduce it to the Standard Form in Figure 2. Note that any two blocks in the diagram, where a single output connects to a single input, can be combined by multiplying their transfer functions together and assigning the result to the combined block.

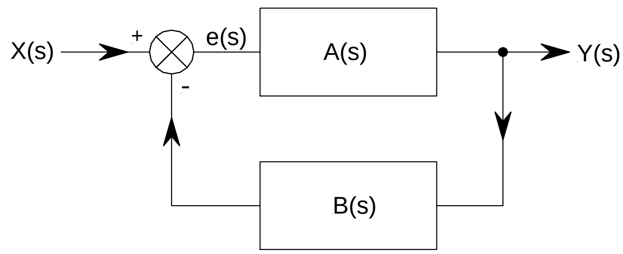

Figure 2 – Standard form of feedback loop

First we need to deal with the Mass Position output. For analysis purposes we want to move it outside the feedback loop. To do that we just have to make a copy of the integrator block.

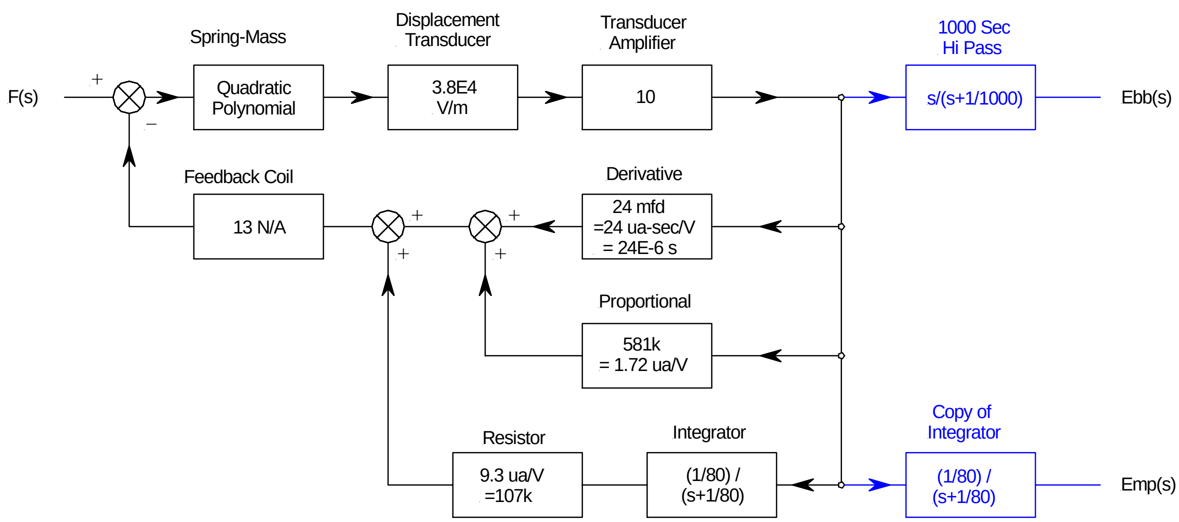

Figure 3 – Integrator duplicated

Finally because the coil resistance is low compared with the three feedback branches, they can easily be combined into a single block with its transfer function being approximately the sum of the transfer functions of the three branches.

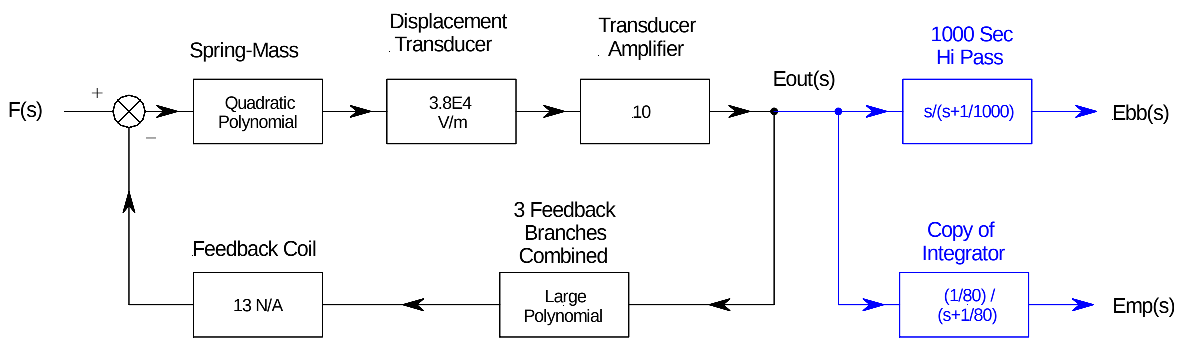

Figure 4 – Feedback branches combined

Now we have a standard form feedback loop with the Spring-mass, the position transducer and its amplifier making up the forward path, and the feedback branches and force-feedback coil in the feedback path. Eout/F(s) is the Closed Loop Transfer Function (*CLTF*), with Eout then being applied to the 80-second high pass filter to obtain the broadband output or to the hypothetical copy of the integrator to obtain the mass position output signal.

One particularly important result of this analysis is: *At all frequencies where the feedback loop-gain is high, the characteristics of the spring-mass, the displacement transducer and its amplifier have minimal effect on the transfer function Eout/F(s).* That transfer function will be determined almost totally by the characteristics of the feedback branches and the force-feedback coil. One important implication of this is that the amplitude response of the overall instrument does not depend on the free period or Q of the spring-mass. In fairness, it should be noted that there are some other advantages to designing a long free period and high Q, but those do not directly affect the closed loop frequency response.

Transfer functions

There are four ways in which a frequency may be expressed. The notation used in the following discussion will be:

1. f frequency - *cycles*/sec ≡ Hz
2. T period ≡ 1/f - sec/*cycle*
3. ω angular frequency = 2π f – *radians*/sec
4. τ time constant ≡ $1/\omega = T/2\pi$ – sec/*radian*

ω and τ are the "natural" units for these calculations. For plotting and other descriptive purposes, ω and τ may be converted to f and T. Note also that since *radians* and *cycles* are dimensionless quantities, both ω and f are properly expressed in units of sec^{-1}, and both τ and T in units of seconds, so we have to be careful to observe which form of frequency is being used in order to get the 2π factor correct.

The symbol s is the complex variable, which gets related to frequency by $s = j\omega$, where $j^2 \equiv -1$

Forward-path functions:

Spring-mass

$$\frac{x(s)}{Force(s)} = \frac{1}{M\,(s^2 + s\,R_0/M + K/M)} \quad \text{meters / Newton}$$

where x(s) = mass position - meters
Force(s) = external force applied to the mass - Newtons
R_0 is the velocity damping term - Newtons / meter/sec
M = effective seismic mass - Kg
K = effective spring constant measured at mass M location = dForce/dx - Newtons/meter

Using other variables:

Substituting the natural frequency, ω_0, where $\omega_0 = \sqrt{M/K}$ - *rad*/sec *This relationship may also be used to compute* K *from measurements of* T_0, where T_0 is the undamped natural period in seconds, noting that $\omega_0 = 2\pi/T_0$.

Substitute the damping factor ζ_0, where $\zeta_0 = \dfrac{R_0}{2\sqrt{(MK)}}$ *This may also be used to compute the value of* R_0 *from measurements of* ζ_0.

so, for the spring-mass we have:

$$\frac{x(s)}{Force(s)} = \frac{1}{M\,(s^2 + s\,2\zeta_0\,\omega_0 + \omega_0^2)} \qquad \text{m/N}$$

Displacement Transducer

3.7648E+4 Volts/meter *DC to several hundred hz. Look for phase shifts at high frequencies.*

Transducer Amplifier

x 10 + input filter terms *Filters not included. Their effect mostly at f > 100 Hz*

r = Total displacement transducer constant = 10 x 3.7648E4 ≅ 3.7E5 Volts/Meter

Total Forward-Path expression - *Spring-Mass and displacement transducer*

$$A_F(s) = \frac{r}{M\,(s^2 + s\,2\zeta_0\,\omega_0 + \omega_0^2)} \qquad \text{Volts / N}$$

where ω_0 is the undamped natural frequency. $\omega_0 = \sqrt{K/M}$ - *rad*/sec

$\omega_0 = 2\pi / T_0$, where T_0 is the undamped natural period in seconds

and ζ_0 is the damping factor. $\zeta_0 = \dfrac{R_0}{2\sqrt{MK}}$.

Auxiliary functions:

VBB output filter – 1000 sec hi-pass

$$\frac{s}{(s + \omega_m)} \qquad R_m = 1E6, \quad C_m = 1000E\text{-}6$$

where $\omega_m = 1/R_mC_m = 0.001$ *rad*/sec.

VBB output amplifier.

$$\frac{(s + \omega_1 + \omega_2)}{(s + \omega_2)}$$

where $\omega_1 = 1/R_1C_1$, $\omega_2 = 1/R_2C_1$, $R_1 = 4.7E5$, $R_2 = 2E6$, $C_1 = 1E\text{-}7$

so $\omega_1 = 21.28$ *rad*/sec. (= 3.39 Hz); $\omega_2 = 5$ *rad*/sec. (= 0.80 Hz)

Feedback Path:

Integral Feedback Branch – *Assumes feedback coil resistance is low vs. 107k Ω*

$$\frac{I_{out}(s)}{V_{in}(s)} \cong \frac{\omega_I}{R_I\,(s+\omega_I)} \quad \text{Amperes / Volt sec} \qquad R_I = 1.07E5,\ Ra = 2E6,\ Ca = 40.2E\text{-}6$$

where $\omega_I = 1/\tau_I = 1/RaCa = 1/80.4$ *rad*/sec

Proportional Feedback Branch - *Assumes feedback coil resistance is low vs. 581k Ω*

$$\frac{I_{out}(s)}{V_{in}(s)} \cong 1 / R_p \quad = 1.72E\text{-}6 \quad \text{Amperes / Volt}$$

where $R_p = 5.81E5$ ohms

Derivative Feedback Branch

$$\frac{I_{out}(s)}{V_{in}(s)} = \frac{sC_d}{(sR_fC_d + 1)}$$

where $C_d = 24.1$ µf and R_f = Feedback coil resistance = 8 Ohms

At frequencies much below 825 Hz, $\frac{I_{out}(s)}{V_{in}(s)} \cong sC_d$ Amperes/ Volt/sec

For this analysis, we will ignore R_f and use the simpler expression, sC_d.

Feedback coil

$$G_n \equiv \frac{Force(s)}{I_{in}(s)} = 12.98 \text{ Newtons /Ampere}$$ *Assumed wide-band.*

Total Feedback path - *Assuming feedback coil resistance is low.*

$$B_F(s) \equiv \frac{Force(s)}{V_{in}(s)} \cong Gn\left(sC_d + \frac{1}{R_p} + \frac{1}{R_I}\frac{\omega_I}{(s+\omega_I)}\right) \quad \text{Newtons / Volt}$$

Expressed in terms of frequency ω_B and damping factor ζ_B

$$B_F(s) \cong \frac{(R_I + R_p)}{R_I R_p}\,\frac{\omega_I}{(s+\omega_I)}\,\frac{(s^2 + s\,2\zeta_B\omega_B + \omega_B^2)}{\omega_B^2} \quad \text{Newtons / Volt}$$

where $\omega_B^2 = \frac{\omega_I}{C_d R_I}\left(\frac{1}{R_p} + \frac{1}{R_I}\right)$ and $\zeta_B = \frac{1}{2}\tau_B\left(\omega_I + \frac{1}{R_p C_d}\right)$

Note that with the proportional feedback removed ($Rp \rightarrow \infty$) these reduce to

$$\tau_B^2 = \frac{C_d R_I}{\omega_I} \quad \text{and} \quad \zeta_B = \tfrac{1}{2}\tau_B\,\omega_I$$

Total loop force-response:

The closed-loop system response to a force is given by

$$\frac{E_{out}(s)}{Force_{in}(s)} = \frac{A_F(s)}{1 + A_F(s)B_F(s)} = \frac{1}{1/A_F(s) + B_F(s)}$$

where E_{out} is the output signal voltage, at the intput to the VBB filter.

The second expression may be slightly easier to manipulate.

Plotting the system characteristics:

For the general closed-loop transfer function given by

$$F(s) = \frac{A(s)}{1 + A(s)B(s)}$$

Wherever the loop gain, A(s)B(s) >> 1 this reduces to $F(s) \cong \frac{1}{B(s)}$

Wherever the loop gain << 1 it reduces to $F(s) \cong A(s)$

This suggests that when graphing A(s) and B(s), if we want their graphs to plot compatibly with F(s), the feedback path should be graphed as its inverse, 1/B(s).

Deriving acceleration and velocity responses from the force response.

We first derived the forward path $A_F(s)$ and feedback path $B_F(s)$ transfer functions with respect to an input force. So the entire transfer function with respect to a force input will be

$$\frac{E_{out}(s)}{Force_{in}(s)} = \frac{A_F(s)}{1 + A_F(s)B_F(s)} = \frac{1}{1/A_F(s)+B_F(s)}$$

where E_{out} is the output signal voltage, at the intput to the VBB filter.

The second form of the equation may be slightly easier to manipulate.

To obtain the Acceleration response we use the relationship Force = M x Accel to get

$$\frac{E_{out}(s)}{M\ Accel_{in}(s)} = \frac{A_F(s)}{1 + A_F(s)B_F(s)}$$

Which makes the Acceleration response,

$$\frac{E_{out}(s)}{Accel_{in}(s)} = \frac{M\ A_F(s)}{1+ A_F(s)B(s)} = \frac{M}{1/A_F(s)+B_F(s)} = \frac{1}{1/MA_F(s) + B_F(s)/M}$$

Now Accel. = d(Velocity)/dt And, since in the s-plane, taking the time derivative corresponds to multiplication by s, we have Accel(s) = s Vel(s)

$$\text{so} \quad \frac{E_{out}(s)}{s\ Vel_{in}(s)} = \frac{M\ A_F(s)}{1+ A_F(s)B_F(s)}$$

which makes the Velocity response

$$\frac{E_{out}(s)}{Vel_{in}(s)} = \frac{s\,M\,A_F(s)}{1+A_F(s)B_F(s)} = \frac{s\,M}{1/A_F(s)+B_F(s)} = \frac{1}{1/sMA_F(s) + B_F(s)/sM}$$

The system functions for a velocity input to the STM-8 are graphed in figure 5. Note that this does not include the effects of the VBB and Mass-Position output filters.

The graphs include A, the forward portion of the loop, made up of the spring-mass system, the displacement transducer and its amplifier. This plot currently ignores the effect of the filters following the synchronous detector, but their effect is rather small, and is in the region above 100 hz. The spring-mass was arbitrarily given a damping factor of 0.1, with an undamped natural frequency of 0.5 Hz. That frequency with a mass of 0.5kg implies an effective spring-constant K = 4.935 N/m. The damping factor determines the sharpness and height of the peak in the A function but will not show up in the CLTF.

The system response to a velocity input, the CLTF of Figure 5, is very similar to the one given by S-T Morrissey in his analysis. In the mid-frequencies the gain is computed here as 1590 V / m/s.

Gain-crossover is at about 37 hz. The CLTF deviates somewhat from 1/B in the vicinity of .01hz, where the loop gain dips to 2. [That is not acceptable and the loop gain must be increased] Above the gain-crossover frequency, the transfer function tracks the gain curve of the forward elements, and they are what determine the high frequency portion of the system response. The computations show a phase-margin of better than 87 degrees, which indicates that the loop, *as described*, is pleasantly far from oscillation. In reality, the components which have been ignored, those acting at the higher frequencies, are precisely those which will most strongly influence the loop stability.

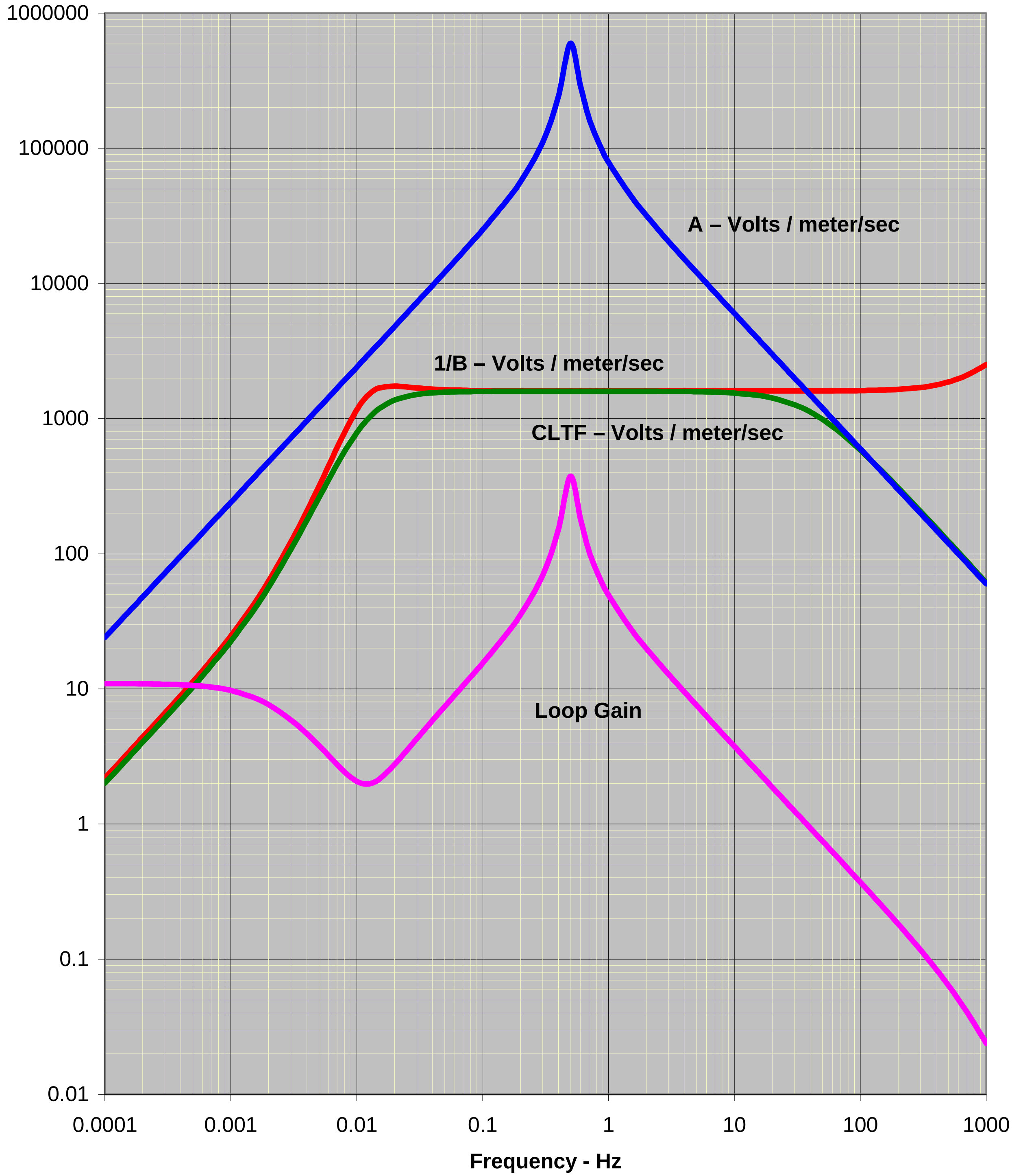

Figure 5 – Loop Functions with respect to velocity – Volts / meter/sec

The input is the velocity of the seismic mass due to ground-motion, and the feedback is the velocity imparted to the seismic mass by the feedback coil. In this log plot it is useful to note that the vertical distance between A and 1/B which = A/(1/B) = AB is the loop gain. The point where A intersects 1/B occurs where AB = 1, which defines the gain crossover frequency.

Another point to be noted here is that the effect of the feedback in shaping the Velocity Response is always to *lower* the response which would exist without feedback (the blue curve). The main effect of adding feedback is to shape a lower and flatter Velocity Response (the green curve). In general, the instrument response with feedback will always be lower than its response without it. Although the low frequency corner with feedback is much lower (0.011 Hz) than the free period of the spring-mass, (0.5 Hz = 2 sec) that is obtained at the expense of sensitivity, though in electronic instruments, sensitivity (gain) is easily added. The main purpose of using feedback is to tightly control the shape of the instrument response.

Evaluating asymptotes (the Bode diagram):

In the log-log graph of figure 5 the various curves appear to consist of straight line segments, connected by relatively shorter curved sections. Extensions of these straight segments are called the asymptotes to the functions, and are shown in Figure 6. We note that all the asymptotes have slopes which are integers. Some are horizontal (slope = 0) some have slope ± 1 (x 10 or x 0.1 per frequency decade, i.e. ±20db per decade), and some have slope ± 2 (x 100 or x 0.01, i.e. ±40db per decade). It is not difficult to compute the equations for the asymptotes. Knowing them is particularly useful when one wants to understand what factors are contributing to what characteristics of the system.

For the system described, the asymptote lines in Figure 6 are:

for the forward function A:

for $0.0001 < f < 0.1$ Hz $\quad A \cong 2\pi f\ M\, r / K$ Volts / m/sec

for $2 < f < 1000$ Hz $\quad A \cong r / 2\pi f$ Volts / m/sec

for the feedback term, expressed as 1/B:

for $f < 0.001$ Hz $\quad 1/B \cong 2\pi f\, M / (G_n(1/R_p + 1/R_I))$ Volts / m/sec

for $0.002 < f < 0.01$ Hz $\quad 1/B \cong (2\pi f)^2\, M\, R_I\, \tau_I / G_n$ Volts / m/sec

for $0.04 < f < 10$ Hz $\quad 1/B \cong M / (C_d\, G_n)$ Volts / m/sec = *velocity response*

The system response (CLTF) ≅ 1/B below 10 Hz and ≅ A above 70 Hz

The asymptotes to the Loop Gain function may be computed for the appropriate frequency ranges by using Loop Gain ≡ AB = A / (1/B) and applying it to the asymptotes for A and 1/B.

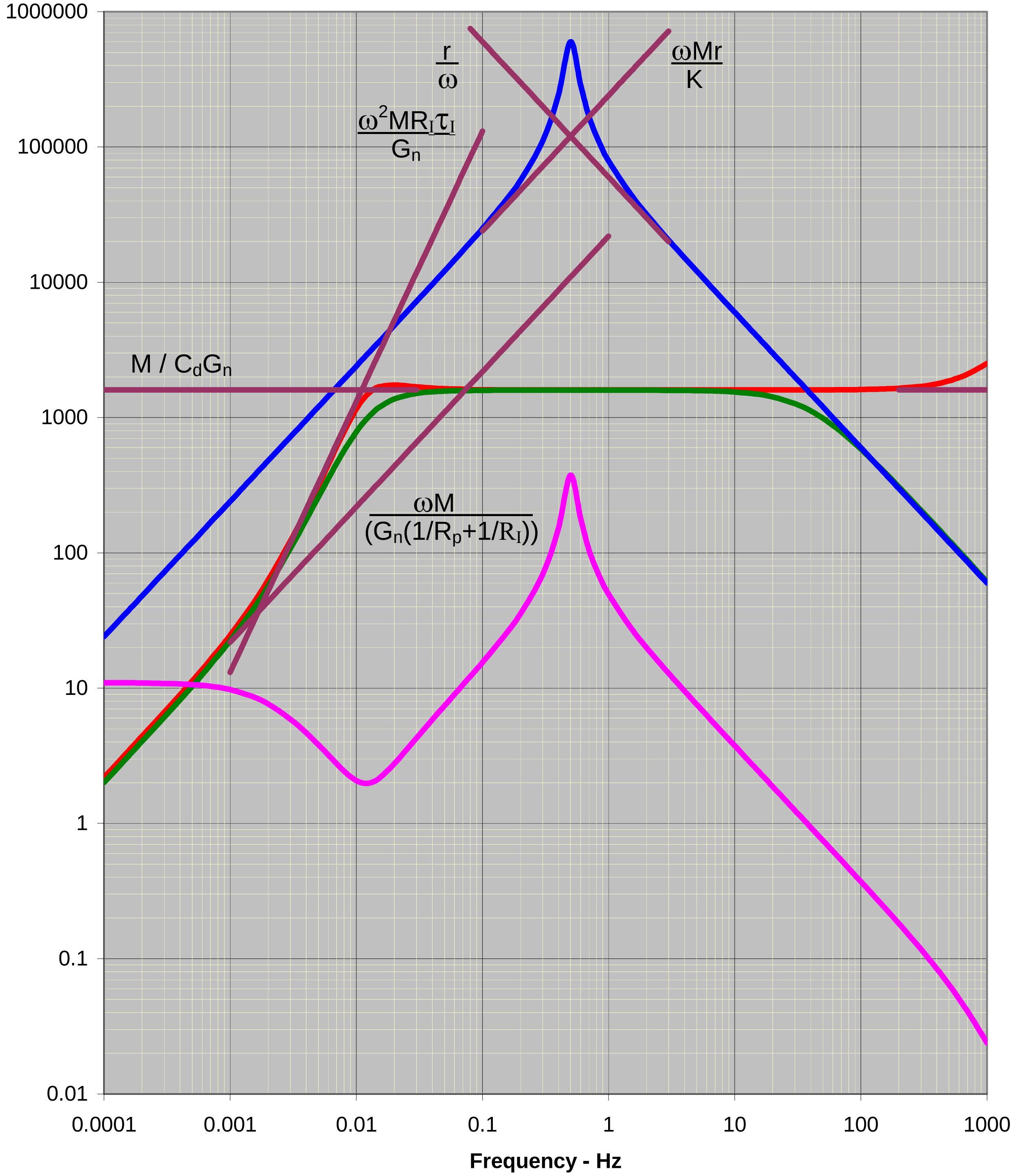

Figure 6 – Loop Functions showing asymptotes

Note that an asymptote slope of +1 is associated with a function which contains ω, and +2 with a function containing ω^2. Slopes of -1 have a $1/\omega$ term and -2 would contain $1/\omega^2$. The horizontal asymptote is a constant and thus has no ω term.

Evaluating Corner Frequencies:

Another useful characteristic of the asymptotes is that they intersect at frequencies which are significant to the loop, i.e. frequencies which correspond to poles or zeros of the transfer function. To find the corner frequencies, the equations for the intersecting asymptotes can easily be set equal and solved for frequency.

• For example equating the rising and falling asymptotes of A shows that they intersect at

$$f = \frac{\sqrt{K/M}}{2\pi} = 0.500\text{ Hz}$$ Which is the natural frequency we assumed for the spring-mass.

• The upward bend of 1/B occurs at $f = \frac{1}{2\pi}\frac{1}{\tau_I\,(R_I/R_p + 1)} = 0.0017\text{ Hz}$

which will complement the 80 sec high-pass filter shape in the Broadband output.

•The low frequency rolloff of 1/B (recall that 1/B ≅ the system velocity response) is at

$$f = \frac{1}{2\pi}\frac{1}{\sqrt{C_d\,R_I\,\tau_I}} = 0.011\text{ Hz}$$ which approximates the natural frequency of the

system with the loop active (90 second period).

• The high frequency rolloff of the system velocity response occurs where A = 1/B which is at $f = \frac{1}{2\pi}\frac{r\,C_d\,G_n}{M} = 37.48\text{ Hz}$ This, it should also be noted, is the gain crossover

frequency.

What's doing what:

From observing the formulas for the asymptote lines and corner frequencies, we can begin to relate individual blocks of the loop to particular portions of the graphs.

P) The proportional feedback path dominates at the very lowest frequencies. The integrator also contributes proportional feedback below its lower cutoff frequency. This is apparent when we observe that the lowest frequency asymptote to 1/B has a term $(1/R_p + 1/R_I)$.

I) The integral feedback path dominates between 0.0017 Hz and 0.011 Hz. Its main benefit appears to to be that it pushes down 1/B at the very low frequencies giving higher loop gain in that region. It is also important in that it gives a 40db/decade slope to the velocity response below the low frequency corner. This is the desired shape, found in many instruments, for the seismometer velocity response.

D) The derivative feedback determines the velocity sensitivity in the main response range, 0.011 - 37 Hz. The transition between integral and derivative feedback at 0.011 Hz results in a quadratic term in 1/B and some possible peaking of 1/B.

Design considerations for VBB vertical seismograph electronics:

1) The aim of the design is to create a force-feedback system which has a desired velocity response, and which is nearly independent of the characteristics of the mechanical spring-mass system. The phase response of the system (essentially, the time delays for different frequencies) should at least be known, and could possibly be deliberately shaped to match other instruments, if waveform correlation will be important.

2) Since this is a feedback system, the loop must not self-oscillate and should not exhibit any resonant peaking at the gain crossover frequency.

3) We want the system to respond to the largest ground velocities expected, without clipping or other waveform distortion, and without creating false long period signals. Low distortion is particularly important if digital post-processing is planned.

4) Finally, any random signals (noise and drift) added by the system should be smaller than the background noise levels at the planned installation site.

Item (1) relates to the design of the feedback (B) elements. After they have been specified, the loop gain AB should be designed to be as large as reasonably possible over the frequency range where we want the system response and distortion to be tightly controlled.

Item (2) relates to how we get rid of all this loop gain, at the higher frequencies. In particular, the phase lag of the loop gain function at the gain crossover frequency (where AB = 1) must not be too close to 180^o. A rule of thumb is that, near gain crossover, the rate of fall of the loop gain should be comfortably less than -2. Also any mechanical resonances above the gain crossover, frequency should not be such that the loop gain peaks back up above 1. This concern will place a practical limit on the instrument's high-frequency response.

Item (3) relates to the clipping levels and nonlinearities of the various loop components. High loop gain helps to linearize the overall loop response. Clipping can occur as a result of either voltage or current limitations in the various sections of the circuit.

Item (4) involves choosing the proper electronic components, designing the circuits to minimize the impact of unavoidable component noise and drift, electrical shielding, supply voltage regulation and probably thermal control.

Each of these four items will likely involve some compromises with the others, and there's unfortunately still the biggest compromise–cost. For portable remote systems, some additional factors which may need to be considered are size/weight, mechanical ruggedness and power consumption.

Appendix I – Variables

Input/Output

E_{out} – Volts	System output voltage (to VBB filter input)
$Force_{in}$ – Newtons	Force on seismic mass due to ground motion.

Spring-Mass

$Accel_{in}$ – meters/sec^2	Acceleration of seismic mass due to ground motion.
$Velocity_{in}$ – meters/sec	Velocity of seismic mass due to ground motion.
M – Kg	Effective value of seismic mass
K – Newtons/meter	Effective spring constant of the spring-mass, measured at the location of M
R_0 – Newtons / m/sec	Velocity damping constant of spring-mass
ζ_0- dimensionless	Damping coefficient of spring-mass
ω_0 – *radians*/sec	Undamped natural frequency of spring-mass

Note: Here, the subscript "0" refers to the spring-mass system.

Circuit elements

r – Volts/meter	Transfer function of the displacement transducer (including the transducer amplifier)
R_p – Ohms = V/A	Proportional feedback resistor
R_I – Ohms = V/A	Integral feedback resistor
C_d – Farads = A/ V/sec	Derivative feedback capacitor

Appendix I - Spring-masses and other second-order systems

A spring-mass system is a second-order system, and as such is defined by two characteristics, its undamped natural frequency and its damping. For a system consisting of a mass M - kg and a spring with a spring constant of K - Newtons/meter, in the absence of any damping, the natural frequency ω_n is defined by the expression $\omega_n^2 = K / M$. So, if the frequency in any of its forms, and either K or M are known, the third variable may be determined.

Real systems also have damping, which may be expressed either as physical velocity damping, R – Newtons / m/s or as a dimensionless damping factor ζ which relates to the time and frequency behavior of the system. ζ and R are related by the expressions:

$R = 2\ K\ \zeta / \omega_n$
$R = 2\ M\ \zeta\ \omega_n$
$R = 2\ \zeta\sqrt{(MK)}$

The value of ζ may be determined experimentally by measuring the amount of peaking in the frequency response of the system, or by observing the time-decaying response of the system to a step disturbance.

Frequently ζ is designed to be $1/\sqrt{2} = 0.707...$ as that gives the sharpest frequency cutoff possible without encountering peaking in the frequency response.

Appendix II – Magnitude computations

This will outline how the magnitude functions used to plot the graphs may be obtained. In reality I used the COMPLEX, IMABS, IMSUM and related Excel "IM" functions to work directly on the functions of $j\omega$. Remember that everything except s and ω are constants.

Spring-mass

$$\frac{x(s)}{Vel(s)} = \frac{s\,M}{K\,(s^2\tau_0^2 + s\,2\zeta_0\tau_0 + 1)} \quad m / m/s$$

where τ_0 is the spring-mass time constant - sec/*rad*
T_0 is the undamped natural period in seconds = $2\pi\tau_0$
and ζ_0 is the spring-mass damping factor.

To get the magnitude of the transfer function we set $s = j\omega$, and indicate magnitudes.
Then we replace the j^2 with -1 and group together the real and imaginary denominator terms.

$$\frac{|x(j\omega)|}{|Vel(j\omega)|} = \frac{|j\omega|\,M}{K\,|(\underbrace{1 - \omega^2\tau_0^2}_{real} + \underbrace{j\omega\,2\zeta_0\tau_0}_{imaginary})|} \quad m/\ m/s$$

Evaluating the magnitudes

$$\frac{x(\omega)}{Vel(\omega)} = \frac{\omega M}{K\sqrt{((1-\omega^2\tau_0^2)^2 + (2\omega\zeta_0\tau_0)^2)}} \quad m/\ m/s$$

Total Forward Path

$$A(s) = \frac{V_{out}(s)}{Vel(s)} = \frac{s\,M\,r}{K\,(s^2\,\tau_0^2 + s\,2\zeta\tau_0 + 1)} \quad \text{Volts/ m/s}$$

$$|A(j\omega)| = \frac{|j\omega\,M\,r|}{|K\,(j^2\omega^2\tau_0^2 + j\omega\,2\zeta\tau_0 + 1)|} \quad \text{Volts/ m/s}$$

$$A(\omega) = \frac{\omega\,M\,r}{K\sqrt{((1-\omega^2\tau_0^2)^2 + (\omega 2\zeta\tau_0)^2)}} \quad \text{Volts/ m/s}$$

Integrator

$$\frac{I_{out}(s)}{V_{in}(s)} \cong \frac{\omega_I}{R_I\,(s + \omega_I)} \quad \text{Amperes / Volt sec}$$

$$\frac{|I_{out}(j\omega)|}{|V_{in}(j\omega)|} \cong \frac{|\omega_I|}{|R_I\,(\omega_I + j\omega)|} \qquad \frac{I_{out}(\omega)}{V_{in}(\omega)} \cong \frac{\omega_I}{R_I\sqrt{(\omega_I^2 + \omega^2)}} \quad \text{Amperes / Volt sec}$$

Derivative Feedback

$$\frac{I_{out}(s)}{V_{in}(s)} = \frac{sC_d}{(sR_fC_d + 1)} \quad \text{Amperes/ Volt/sec}$$

$$\frac{|I_{out}(j\omega)|}{|V_{in}(j\omega)|} = \frac{|j\omega C_d|}{|(1 + j\omega R_f C_d)|} \quad \text{Amperes/ Volt/sec}$$

$$\frac{I_{out}(\omega)}{V_{in}(\omega)} = \frac{\omega C_d}{\sqrt{(1 + \omega^2 R_f^2 C_d^2)}} \quad \text{Amperes/ Volt/sec}$$

If R_f is ignored (its effects are mainly at higher frequencies)

$$\frac{I_{out}(\omega)}{V_{in}(\omega)} \cong \omega C_d \quad \text{Amperes/ Volt/sec}$$

Total Combined Feedback expression - *Assuming feedback coil resistance is low.*

$$B(s) \cong \frac{G_n}{M}\,\frac{(R_I + R_p)}{R_I\,R_p}\,\frac{1}{s}\,\frac{\omega_I}{(s + \omega_I)}\,(\,s^2\,\tau_B^2 + s\,2\zeta_B\tau_B + 1) \quad \text{meters/sec / Volt}$$

where $\tau_B^2 = \frac{C_d\,R_I}{\omega_I}\,\frac{R_p}{(R_p + R_I)}$ and $\zeta_B = \frac{1}{2}\tau_B\,(\omega_I + \frac{1}{R_p\,C_d})$

$$|B(j\omega)| \cong \frac{G_n}{M}\,\frac{(R_I + R_p)}{R_I\,R_p}\,\frac{1}{|j\omega|}\,\frac{\omega_I}{|(\omega_I + j\omega)|}\,|(\,j^2\omega^2\tau_B^2 + j\omega\,2\zeta_B\tau_B + 1)| \quad \text{meters/sec / Volt}$$

$$B(\omega) \cong \frac{G_n}{M}\,\frac{(R_I + R_p)}{R_I\,R_p}\,\frac{\omega_I\sqrt{(1-\omega^2\tau_B^2)^2 + (\omega\, 2\zeta_B\tau_B)^2}}{\omega\sqrt{(\omega_I^2 + \omega^2)}} \quad \text{meters/sec / Volt}$$

Output filter – 1000 sec hi-pass

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s}{(s + 1/\tau_P)} = \frac{s}{(s + \omega_p)} \qquad R_p = 1E6, \qquad C_p = 1000E\text{-}6$$

where $\tau_P = R_pC_p = 1000$ sec/*rad*. $\omega_p \equiv 1/\tau_p = 0.001$ *rad*/sec

$$\frac{|V_{out}(j\omega)|}{|V_{in}(j\omega)|} = \frac{|j\omega|}{|\omega_p + j\omega|} \qquad \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\omega}{\sqrt{\omega^2 + \omega_p^2}}$$

Appendix III. – Variables

In general:

f	frequency - *cycles*/sec ≡ Hz
T	period ≡ 1/f - sec/*cycle*
ω	angular frequency = 2π f - *radians*/sec
τ	time constant ≡ $1/\omega = T/2\pi$ - sec/*radian*

Note: Some sources also use T for time constants, but I prefer to reserve T to represent the oscillation period.

Mechanical:

x(s) =	mass position - meters
$Force_{in}(s)$ =	External force applied to mass due to ground motion - Newtons
R_0	Velocity damping term - Newtons / meter/sec
M =	effective seismic mass - Kg
K =	effective spring constant measured at mass M location = dForce/dx - Newtons/meter
ω_0 =	natural frequency of spring-mass - radians/sec
ζ =	damping factor of spring-mass

Transfer Functions:

$A_F(s)$	Transfer function of forward branch of feedback loop
A(s)	Transfer function of forward portion of feedback loop.
B(s)	Transfer function of feedback portion of loop.
ζ_B	Effective damping factor of B(s).
ω_m	Corner frequency of VBB output filter - radians/second
ω_I	Corner frequency of integral feedback branch - radians/second
ω_B	
ω_1	
ω_2	

Component Values:

R_m	VBB output filter resistor - Ohms
C_m	VBB output filter capacitor - Farads
R_1	
R_2	
C_1	
R_I	Value of resistor in integral feedback branch - Ohms
Ra	
Ca	
R_p	Value of resistor in proportional feedback branch - Ohms
C_d	Value of capacitor in the derivative feedback branch - Farads
R_f	
G_n	Feedback-force transducer constant - Newtons/Ampere
r	Displacement transducer + amplifier constant - Volts/meter

Variables used by S-T Morrissey:

The transfer function:

a: Here is the formulation of the transfer function; the parameters are similar to those in the thesis of J. Steim and the Wielandt and Steim studies where:

For the mechanical system:

- M = sensor effective mass - kg
- T_0 = Mechanical free period of the sensor - seconds (range 1 -15 sec.)

For the feedback system:

- r = displacement transducer constant - volts/mm (range: 80 - 1000 V/mm)
- G_n = force transducer constant - newtons/amp (range 10 -100 N/A)
- G = force transducer constant - m/sec^2/amp
- C = feedback capacitor - farads (range 1 - 36 uf)
- R_p = Proportional feedback resistor - ohms (range 100 - 1000k)
- R_I = Integral feedback resistor - ohms (range 200 -2000k)
- R_F = force transducer resistance - ohms (range 50 -500 ohms)
- T_I = Integrator time constant - seconds (range 10 - 1000 seconds)
- T_d = Displacement transducer time constant (less than 0.06 sec)

For the over-all instrument:

- T_n = Effective natural period - seconds (range 20 to 600 sec)
- ζ = Instrument damping factor

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Following to be deleted?

In terms of acceleration, we use Force(s) = M Acc(s)

$$\frac{x(s)}{Acc(s)} = \frac{1}{(s^2 + s\,2\zeta\,\omega_0 + \omega_0^2)} \qquad m / m/s^2$$

Expressing A(s) in terms of acceleration:

Acc(s) = dVel(s)/dT = s Vel(s) *Multiplying by s corresponds to differentiation*

$$\frac{x(s)}{Vel(s)} = \frac{s}{(s^2\,\tau_0^2 + s\,2\zeta\tau_0 + 1)} \qquad m / m/s$$

where Vel(s) = velocity of the mass - m/s

Expressed in terms of velocity,

$$A(s) = \frac{s\,M\,r\,\omega_0^2}{K\,(s^2 + s\,2\zeta\,\omega_0 + \omega_0^2)} \qquad \text{Volts/ m/s}$$

Expressing B(s) in terms of acceleration

$$B(s) \equiv \frac{Accel(s)}{V_{in}(s)} \cong \frac{Gn}{M}\left(sC_d + \frac{1}{R_p} + \frac{1}{R_I}\frac{\omega_I}{(s + \omega_I)}\right) \qquad m/s^2 \text{ / Volt}$$

In terms of velocity

$$B(s) \equiv \frac{Vel(s)}{V_{in}(s)} \cong \frac{Gn}{M}\,\frac{1}{s}\left(sC_d + \frac{1}{R_p} + \frac{1}{R_I}\frac{\omega_I}{(s + \omega_I)}\right) \qquad \text{meters/sec / Volt}$$

Expressed using time constant τ_B and damping factor ζ_B

$$B(s) \cong \frac{G_n}{M}\,\frac{(R_I + R_p)}{R_I\,R_p}\,\frac{1}{s}\,\frac{\omega_I}{(s + \omega_I)}\,(s^2\,\tau_B^2 + s\,2\zeta_B\tau_B + 1) \qquad \text{meters/sec / Volt}$$

To compute the acceleration response, the expressions for A(s) and B(s) derived above may be used in the above equation.

It should be noted that *for those frequencies where loop gain AB is high*, the closed loop transfer function $V_{out}(s)/Force_{in}(s) \cong 1/B(s)$ Volts/Newton, where $V_{out}(s)$ is the output signal voltage and $Force_{in}(s)$ the the input force on the mass M, due to ground motion.