

# The use of feedback in seismic sensors.

Brett Nordgren

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brett3nt@bnordgren.org

There is a lot of confusion about the role of feedback in the design of seismic sensors. This is an attempt to clarify exactly what feedback can and cannot accomplish when applied to such devices and to suggest how to go about designing a feedback sensor.

All such sensors have a seismic mass coupled to some form of position or velocity transducer which produces an electrical signal in response to the apparent motion of the mass. With care, such transducers can be constructed to be quite sensitive and relatively free from internally generated noise. For vertical instruments, the mass is supported by some form of spring. For horizontal instruments, some form of pendulum is most common, with gravity providing the restoring force.

The difficulty comes when we want to tightly control the frequency response of such a device, or equally important, accurately know its phase response or time delays over the band of frequencies of interest, which is essential to do if its data are to be compared with data from other instruments. Another difficulty comes when we try to maintain the proper centering of the mass in the presence of slow changes in the device or its surroundings. These could arise from changes in temperature, slow changes in ground tilt, earth tides, or in the case of a vertical instrument, spring creep, as well as from numerous other potential sources. In a sensitive instrument such changes could be great enough to move its output completely out of range before mechanical adjustments can be made. Feedback, properly applied, can be used both to shape the instrument response and also to counter some of the effects of slowly-applied errors. Finally, feedback will have the effect of greatly reducing the motion of the mass in response to seismic ground motion. This means that with feedback we might be able to use a displacement transducer which has quite a small range of operation, but which, in return, could be very sensitive. In addition, by limiting the sensor motion we can greatly reduce the effect of transducer and other system nonlinearities. It should be noted that we will be looking here at a feedback system which senses the apparent position of the seismic mass and then feeds back a signal which is used to apply a force to the mass to counter any changes.

The frequency response of a seismic sensor may be described in more than one way. Most commonly, instruments are described by a mathematical equation representing their response to ground velocity. Starting with the spring-mass alone, we can plot its velocity response (Fig. 1). Now, if the velocity curve is integrated with respect to time we get a curve representing instrument response to a position or displacement input (Fig. 2). If the velocity curve is differentiated with respect to time, a curve is obtained which represents the response to an acceleration (Fig.3) or, equivalently, to a force (Fig. 4). These curves are just four different ways of describing the same device.

The graphs show the response curves for a spring-mass such as might be used in a vertical seismometer. In general, the parameters used here are those used in Sean-Thomas Morrissey's STM-8 vertical seismometer design. The mass,  $M$  was assumed to be 0.5 kg supported by a spring having an effective spring constant,  $K$  of 4.935 N/m., giving a natural period  $T_0$  of about 2.0 seconds or natural frequency,  $F_0$ , of 0.5 Hz. A damping coefficient,  $\zeta$  of 0.1 was assumed. This choice is arbitrary, for as we will see, the assumed damping coefficient will have little effect on the instrument response we obtain after feedback has been added.

## Spring-Mass Velocity Response

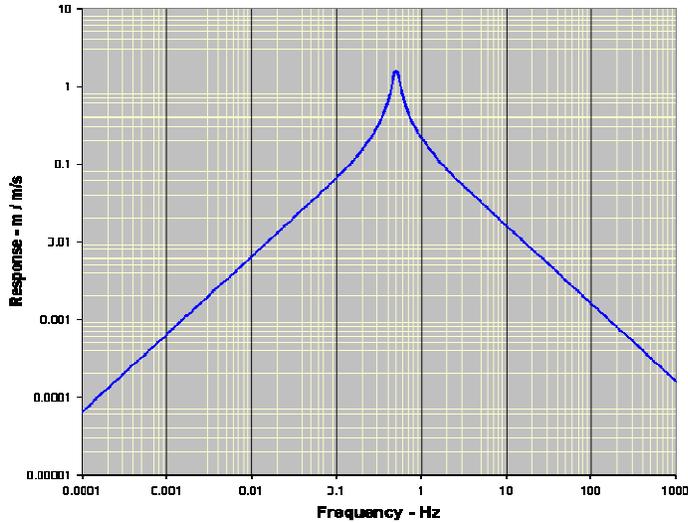


Figure 1

For ground motion described by velocity, the displacement of the spring-mass reaches a peak at 0.5Hz, and falls on either side of that frequency. With a little more damping, this could become the basis for a respectable narrow-band velocity sensor, centered at 0.5 Hz.

## Spring-Mass Position Response

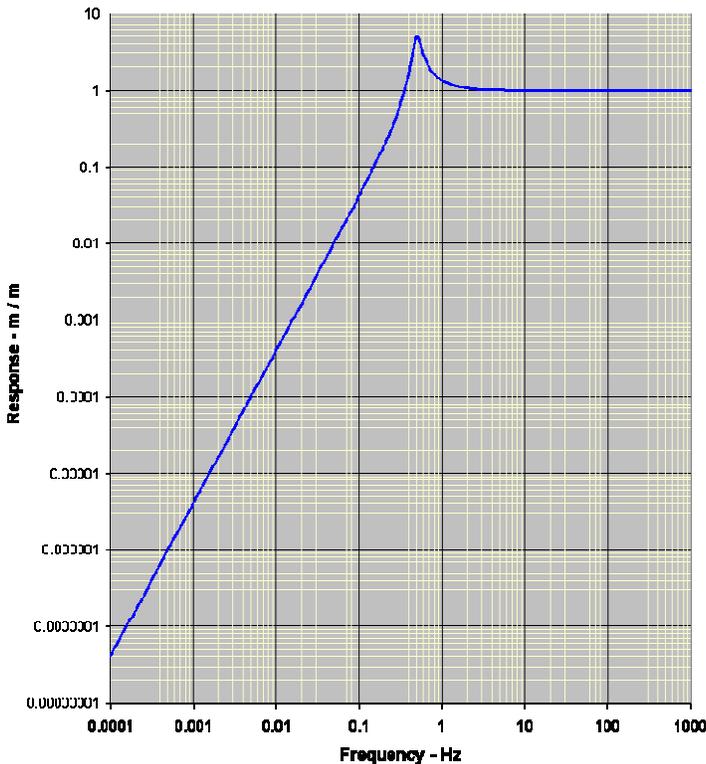


Figure 2

A position sensor attached to the mass, would see essentially 100% of ground displacement frequency components above 1Hz. For components below 0.5Hz, the response falls rapidly with decreasing frequency, being down to about 4% at 0.1Hz.

### Spring-Mass Acceleration Response

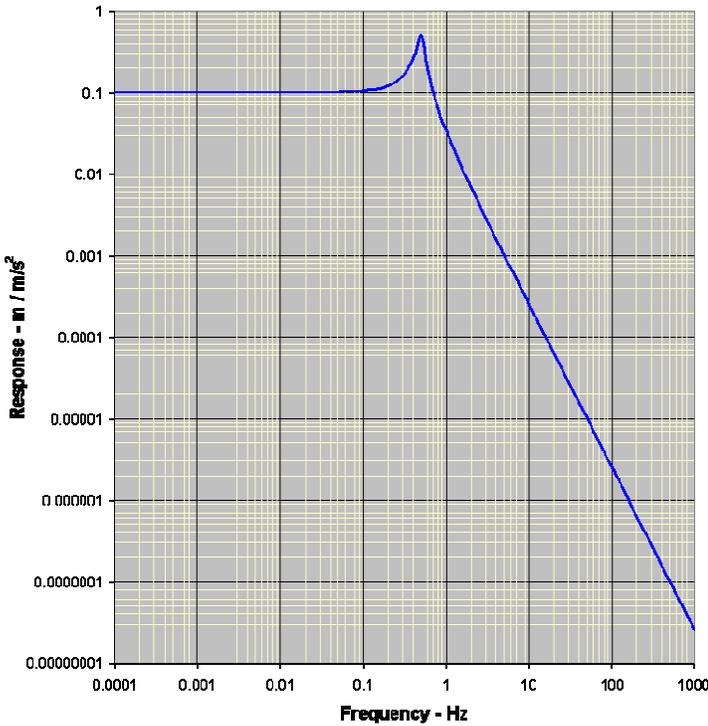


Figure 3

For ground motions described as accelerations, at low frequencies the response is constant, a little over 0.1 meter / m/sec<sup>2</sup>.

To see where that comes from, we can recognize that at low frequencies, the spring-mass response is a function of the spring characteristic as it responds to a slowly varying acceleration force on the mass. We can use the relationship  $F=MA$  and the definition of our spring constant  $F=KX$  to derive  $X/A$  which is what we are plotting here. At low frequencies, the response is constant, equal to  $M/K$ , or  $0.5\text{kg}/4.935\text{ N/m} = 0.1013\text{ m / m/sec}^2$ .

### Spring-Mass Force Response

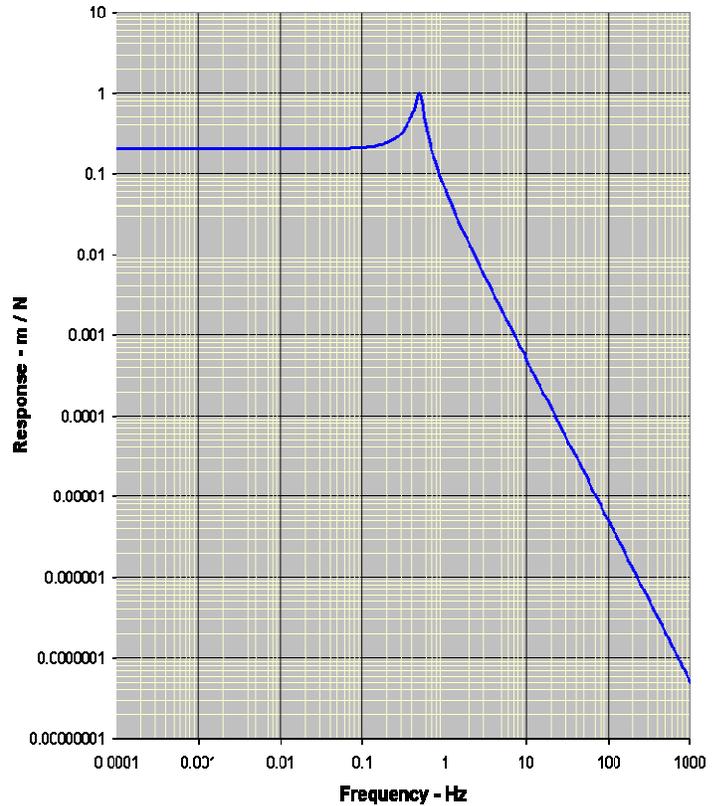


Figure 4

If we want to examine the spring-mass response to a force, we see that the response curve is similar to that for acceleration, and is related to it by simply having been divided by the mass, 0.5kg. At low frequencies, the response is a constant 0.2026 m/N. Note that this is simply the reciprocal of the spring constant of 4.935 N/m.

In the above example let us look at the effect of a small, slow, temperature change on the spring. If the temperature of a steel spring increases by 1deg C, its spring constant will decrease by about 0.02%. The mass exerts a steady force of  $0.5\text{Kg} \times \sim 0.9802\text{ m/sec}^2 = 0.490\text{N}$  which is exactly balanced by the spring. A 0.02% reduction in K will result in motion of  $-0.49\text{N}/0.4935\text{ N/m} \times 0.02\%$  or roughly -0.2mm. By seismometer standards, this is a rather large motion, and a temperature change of only a few degrees might even challenge the range of the displacement sensor. The problem with the system described here, is that its response to an error-induced force, even one applied extremely slowly, is rather large.

## How can feedback help?

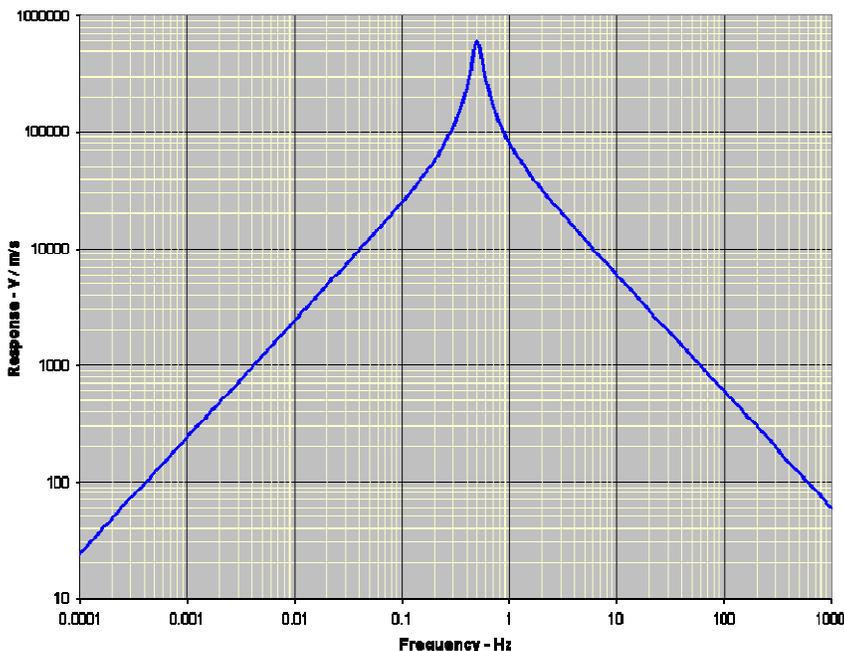
It was said above that the purpose of adding feedback will be to shape the frequency response of the instrument. First, we observe that feedback can not affect the size of the error-induced signals by themselves. For example it can not affect only the force-error resulting from the  $0.02\%/^{\circ}\text{C}$  decrease in the spring constant. However feedback *can* be used to shape the frequency response of our instrument. If we could use feedback to cause the instrument's response to *any* force to fall toward zero as one approached zero frequency, then its response to a slowly applied error force would also approach zero. In such a case, a *sufficiently slow* change in the spring force would not move the mass position (much) and would not show up in the output. This then changes our design problem, from one of largely eliminating errors to one of just controlling the rate of change of each error to make it sufficiently slow—possibly an easier task.

Looked at another way, if we can set a lower limit to the seismic frequencies which we want to detect, we can use feedback to reduce the instrument's response to *both* seismic and internal “noise” inputs which are below that frequency. Conversely, for frequencies which are within the frequency band we want to observe, the ratio of seismic signal to instrument noise will never be changed by adding feedback.

## Shaping the response:

The application of feedback can be thought of as working to carve away the initial sensor response until it matches the shape we desire. This process can be likened to what was reportedly said by the sculptor, Michelangelo. When asked how he proposed to sculpt an angel out of a large block of marble, his annoyed reply was something like “It's extremely simple, I will just remove everything that's not the angel.” Similarly, feedback simply carves away everything that's not the response we want. In any case, the application of feedback generally acts to *reduce* the sensitivity of the initial device, just as the marble boulder must be reduced to make the angel. It should be noted that obtaining the sensitivity we need to start with is not difficult, as amplifier devices with high gain are easily obtained. The practical challenge will be to maintain low noise.

### Spring-Mass + Sensor Velocity Response



We will start by attaching a position transducer to the mass and connecting its output to an amplifier. This provides a voltage signal which builds up the velocity response and gives us something to start carving on. The gain factor we choose for the amplifier will depend, first, on the mid-frequency velocity response we want to obtain in the final design, and, second, on the high-end frequency where we want the instrument response to begin to fall off. Assuming we want to get a mid-band velocity sensitivity of 1640 Volts/ meter/sec with a high-frequency rolloff of 35.91Hz (both from the STM-8) we will need to design the sensor gain to be  $1640 \times 35.91 \times 2\pi = 370000 \text{ V/m}$ , which value is assumed here to be constant at all frequencies of interest. (Though in practice it will not be at the highest frequencies.)

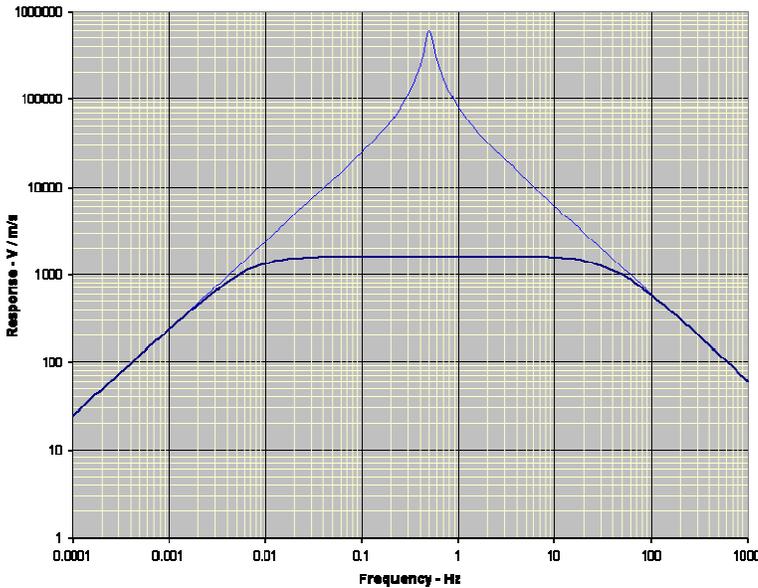
Figure 5

**Now comes the feedback:**

We can now add a force-feedback coil, having the force-constant of the STM-8 of 12.98 N/A. Then, we select the derivative-feedback capacitor to achieve the mid-band velocity sensitivity we want. The capacitor value in Farads will be = Mass / (Desired sensitivity x force-constant) Again, using the STM-8 numbers,

$$C_d = \frac{0.5\text{Kg}}{1640\text{V/ m/sec} \times 12.653 \text{ N/A}} = 24.1\text{E-6 Fd or } 24.1\mu\text{F}$$

**Derivative Feedback only**  
**Velocity Response**

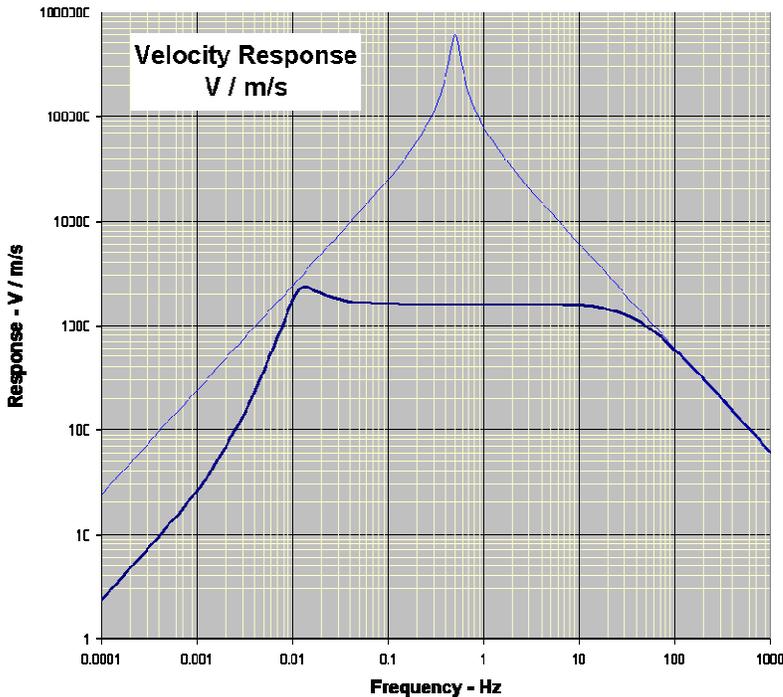


**Figure 6**

Adding derivative feedback has carved away the top of the original (light blue) velocity response to give a flat response curve at 1640 V / m/s from about 0.007 Hz to 35.9 Hz.

We also can see that the assumed damping factor for the original spring-mass really doesn't matter once the derivative feedback has carved down the response. The spring-mass damping could have been much smaller (higher peak) or much greater (more rounded peak) and the Velocity Response curve with derivative feedback would still look exactly the same.

From here on, the right, high-frequency, shoulder of the response curve will be left alone. As you go above 35.9 Hz the effect of the feedback disappears, leaving just the original spring-mass response.



**Figure 7**

But we *will* be carving on the low-frequency shoulder by adding an integrator branch to the feedback path. In the STM-8, the integrator is approximated by an R-C circuit, 2.0MΩ - 40.2μf. This R-C is followed by a unity-gain buffer amplifier, which feeds a scaling resistor R<sub>I</sub> = 107kΩ which determines the strength of the integral feedback. As with all R-C integrator approximations, this one only integrates at frequencies above its low frequency corner, here located at 1/(2πRC) = 0.00198 Hz. Below that frequency the integrator branch acts as proportional feedback. We see that after adding the integrator the low-frequency corner of the velocity response has moved slightly higher, to 0.01 Hz, and also there is now a modest peak at that frequency. The peak is due to an electrical resonance created where the derivative and integral feedback branches interact.

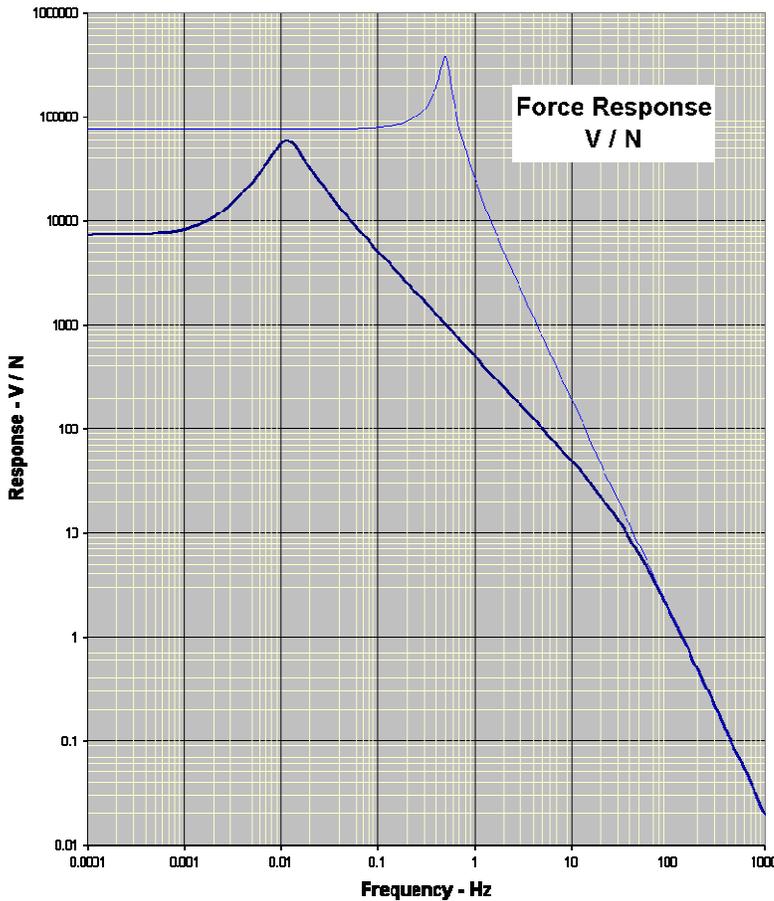


Figure 8

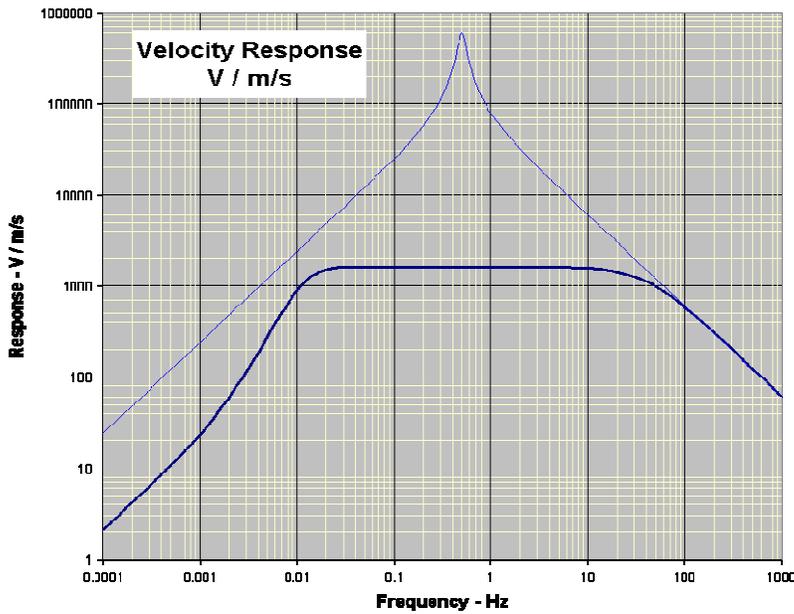


Figure 9

It may also be helpful to look at what we now have with respect to the force response. At frequencies below 0.01 Hz, the integrator reduces the force response, so that below 0.001 Hz it has been reduced from about 75,000 V/N down to 7,600 V/N, which means that the instrument is almost 10 times less sensitive to slowly applied forces than it was without the integral feedback. Ideally we would like the force response to continue to fall with decreasing frequency, so that at DC its sensitivity to all forces, including error-induced forces, approaches zero. This requires designing an integrator which continues to be effective at ever lower frequencies. To do that would likely require significant change to the integrator circuit design, along with the use of a better quality (polypropylene) integrating capacitor.

Now, to clean up the bump at 0.014Hz in the velocity response, we will add a bit of proportional feedback. This adds damping to the resonance introduced by the integral-derivative feedback combination. Here the proportional feedback resistor,  $R_P$  is 750k, slightly larger (less feedback) than the 561k used in the STM-8, but 750k gives a better (sharper) shape to the Velocity Response low-frequency rolloff.

### **Dealing with the lowest frequencies:**

What we have designed so far still has a substantial compliance to slowly applied forces (6,738 V/N or ~18 mm/N). Ideally, we would want the force response to approach zero for forces which were applied infinitely slowly.

The velocity responses of most commercial broad-band seismometers are described as having two (or even three) s-plane zeros at zero frequency. Two or more zeros at zero in the velocity response implies that the instrument's response to any force is zero at zero frequency. We would like this to be true in our design also.

Having two zeros at zero means that the low end of the velocity response curve continuously falls, by a factor of 100 for each decade decrease in frequency, commonly described as 40db per decade. Our curve does so, but only down to about 0.002Hz. It only has one zero at zero. Instead, we would like to see the 40db per decade slope continue down indefinitely far.

### **Hiding the drift:**

There are two approaches to achieving this. As is done in the STM-8, we can make the instrument *appear* to not respond to zero-frequency forces. To do that, we can add a high-pass filter to the instrument output, set to cut off below 0.002Hz, designed to be complementary to the low frequency break in the velocity response near 0.002Hz. This is the cosmetic approach. A voltmeter on the filter output will not register any significant response to a very slowly applied force. However, when using this approach, the spring-mass will still be moved, by 18 mm per Newton, and could still be pushed entirely out of its sensor range in response to a rather small but steady error force.

### **Building a better integrator:**

The second approach is to modify the low end frequency response as before, but to do it, instead, by modifying the feedback loop. Then the system will truly be able to resist slow forces. To do this requires that the low-frequency corner of the integrator branch be made as low as possible. There is a limit to how far one can go, but perhaps even 0.8  $\mu\text{Hz}$  might be possible using the best quality polypropylene capacitors (200,000 sec. leakage time-constant) along with a very good (very low input current) op-amp. If we design the integrator to work down to 10  $\mu\text{Hz}$ , the DC force compliance of our design could be reduced to 0.12 mm/N. That means that a force of 0.1 N would only move the mass by 0.012mm.

But not really. The dynamic range of the feedback paths, particularly the integrator, would have to be much greater. To resist 1/10 N you would need to supply almost 8 ma to the feedback coil. The integrator branch would need to be able to supply this current, but in the original design it can only provide a few tens of microamperes. Some form of buffer current-amplifier would need to be added in order to take advantage of the better integrator.

A promising alternative approach to creating a very low frequency integrator might be to do the integration digitally, followed by a D/A converter to provide the integral feedback signal. This approach is being investigated further.

## How strong is the feedback?

We should start by addressing the two questions, how do you define feedback strength, and why does it matter? Feedback strength varies with frequency, and at any particular frequency it is defined by the “loop gain”. Loop gain is closely related to how deeply we have carved down into the original Spring-Mass+transducer+amplifier response to arrive at our feedback-controlled response. In Figure 9, at 0.01 Hz, for example, the loop gain is directly related to the vertical distance between the dark blue curve and the original light blue one. Since this is a log graph, that distance represents a ratio, which at 0.01 Hz is about 2.62:1. The loop gain equals the ratio-1, so at 0.01 Hz it equals 1.62.

The main purpose of using feedback is to create a device whose response depends entirely on the Proportional, Integral and Derivative feedback branches and not at all on the characteristics of the spring-mass, position transducer and amplifier, on the assumption that the feedback branches can be designed to be much more accurate, linear and noise-free than the original device without feedback. Loop gain determines to what degree the feedback branches are in control. If, at a particular frequency, we want the instrument response to be 99% determined by the feedback branches and only 1% by the original spring-mass and transducer, the loop gain there would need to be 100. Feedback designers generally are happy when they can work with loop gains of 100 or more. And this is also a justification for our wanting to carve on the lower frequency shoulder of the original response. By carving deeply we assure that the resulting performance at the lowest frequencies is dominated by the feedback elements and not the sensor and spring with their nonlinearities and noise. Curiously, here is an improvement which does not depend on the frequency response curve which we have carved out with feedback, but rather on the size of the block we started carving on. By using high transducer gain at the lower frequencies we can improve the force-noise rejection at those frequencies without affecting the instrument response.

I would note that in the STM-8 as designed, the loop gain is not adequate in the 0.001 to 0.06Hz region, throughout which it is below 10 and at places, below 2. A possible cure would be to add some additional gain, perhaps 70, to the transducer amplifier over the frequency range from DC up to about 0.015Hz, and from there arrange for the added gain to fall back to unity by 1Hz. If you do that, along with using the better integrator described above, the compliance at 10uHz drops down to 1655 nm per Newton. So, if a steady force was applied to the boom, it would soon manage to return to almost precisely the same level as before the force.

### Overdamping the spring-mass:

As an experiment, let us see what the velocity response of the spring-mass, transducer, amplifier combination looks like if we add some serious mechanical damping without using any feedback. Doing something like this has been discussed elsewhere in connection with broadening the frequency response of a geophone, but here we will try it with the STM-8 spring-mass.

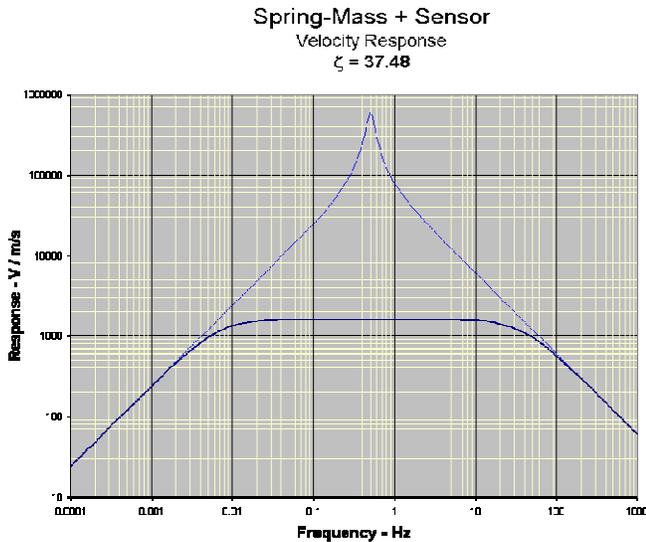


Figure 10

The damping we want to add will be velocity damping, in which the damping force is proportional to the velocity of the mass.

Without using any feedback, here we have applied damping of 117.7 N/ m/s, which is equivalent to a damping factor,  $\zeta = 37.48$

The point here is that when we sense the displacement of the mass and convert it to a voltage, then differentiate that voltage to create a current proportional to velocity and finally use that current to apply a force to the mass to oppose its motion, we are achieving exactly the same thing as mechanical velocity damping. Our derivative feedback branch is nothing more than a big velocity damper.

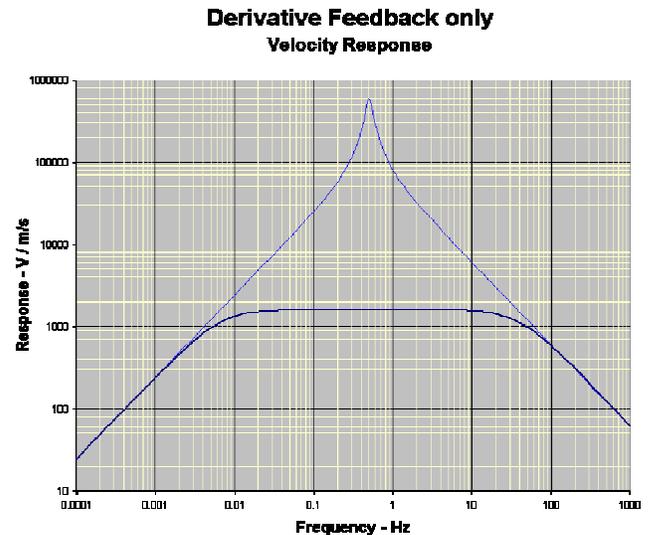


Figure 11

This is the velocity response as before, using only derivative feedback. The response curve is essentially identical to the velocity response obtained when we simply overdamped the spring mass, without adding any feedback.

Now for some math:

Using the parameters from S-T Morrissey's STM-8 vertical:

M – seismic mass	0.5 kg
K – spring constant	4.935 Newtons/meter
$\zeta$ – spring-mass damping factor (assumed)	= 0.1
r – displacement transducer sensitivity	37000 Volts/meter*
$G_n$ – feedback coil force constant	12.653 Newtons/Ampere
$C_d$ – derivative feedback capacitor	24.1 $\mu$ Fd
$R_p$ – proportional feedback resistor	561K $\Omega$ *
$R_I$ – integrator scaling resistor	107K $\Omega$ *
$\tau_I$ – integrator time constant	80.4 seconds*
V / dx/dt – instrument velocity sensitivity	1640 V / m/s

\* Values from the original design which were later changed to improve performance.

### Damping:

Three approaches can be used to mathematically define the damping of the spring-mass.

The first is the mechanical damping coefficient R, with units of Newtons / meter/second.

Alternatively, the damping can be described by a dimensionless damping factor, sometimes called the damping ratio, here represented by the Greek letter zeta,  $\zeta$ .

Another dimensionless measure of damping, the quality factor, Q, is most commonly used in connection with resonant electrical circuits.

These are related to each other as follows:

R, the mechanical velocity damping term – Newtons / m/s

$$\zeta, \text{ the damping factor – dimensionless} = \frac{R}{2\sqrt{(MK)}} = \frac{R}{2M\omega_0} = \frac{R\omega_0}{2K} = \frac{1}{2Q}$$

$$Q, \text{ the quality factor – dimensionless} = \frac{\sqrt{(MK)}}{R} = \frac{K}{R\omega_0} = \frac{M\omega_0}{R} = \frac{1}{2\zeta}$$

where the natural frequency,  $\omega_0 = \sqrt{(K/M)}$

Frequency constants:

$f_0$  – Hz the natural frequency of the spring-mass = 0.5 Hz ( $\text{sec}^{-1}$ )

$\omega_0$  – Natural frequency - radians per sec, or just  $\text{sec}^{-1} = 2\pi f_0 = 3.1416 \text{ sec}^{-1}$

Frequency variables

f – cycles/sec, Hz or just  $\text{sec}^{-1}$

$\omega$  – Radians per sec or  $\text{sec}^{-1} = 2\pi f$

Time variables

$T$  – period =  $1/F$  seconds/cycle or just seconds

$\tau$  – time constant =  $T/2\pi$  seconds/radian or just seconds

Since the units for  $\tau$  and  $T$  and for  $\omega$  and  $f$  are the same, it is often left up to the reader to determine which is meant, and to decide whether the  $2\pi$  factor is required.

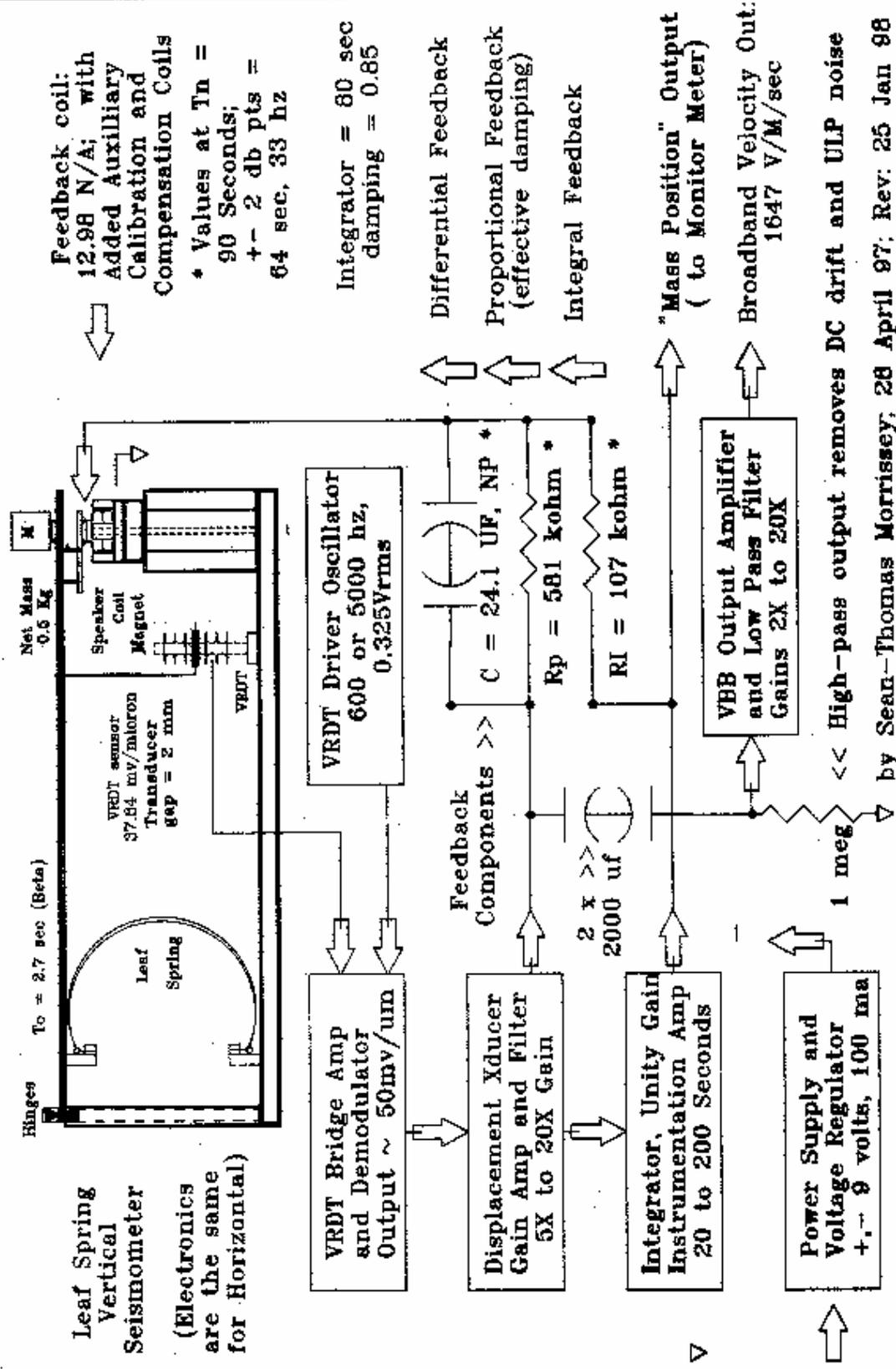
In our formulas we will often use the  $\omega$  form for frequency as it avoids many  $\pi$ 's.

**Observation:**

Applying derivative feedback is exactly the same as applying a mechanical damping factor to our spring-mass. The equivalent velocity damping term,  $R = G_{nr}C_d$  Newtons / m/s.

That value of  $R$  is the same as having a damping coefficient,  $\zeta = G_{nr}C_d/(2M\omega_0)$

# Block Diagram of the Electronics for the Leaf Spring Broadband Seismometer, using a VRDT Displacement Transducer and the Coil and Magnet of a Large Speaker for the Feedback Transducer.



by Sean-Thomas Morrissey; 28 April 97; Rev: 25 Jan 98

**Transfer Function of a Broadband Feedback Seismometer.  
Parameters for a LOW-Z. SPEAKER SEIS. VBB config.**

Values: for leaf-spring sensor with 8-ohm speaker coil

$m_v := \text{volt}^{-3} \cdot \mu = \text{m}^{-6}$   
 $i := 1, 2, \dots, 600$   
 $f_i := 10^{(i-300) \cdot 0.01}$   
 $\omega_i := 2 \cdot \pi \cdot f_i$   
 $s_i := j \cdot \omega_i$   
 $s_{200} = 0.6283i$

$r := \text{displacement transducer constant, volts/meter}$   
 $G := \text{force transducer constant, newtons/amp}$   
 ..... function requires m/sec/sec/amp  
 $M := \text{sensor mass, kg}$   
 $C := \text{feedback capacitor, farads}$   
 $R_p := \text{Proportional (DC) feedback resistor, ohms}$   
 $R_I := \text{Integrator feedback resistor, ohms}$   
 $R_F := \text{force transducer coil resistance}$   
 $T_I := \text{Integrator time constant, seconds}$   
 $T_0 := \text{Mechanical free period of sensor, seconds}$   
 $T_d := \text{Displacement Xducer time constant } (\sim= 0)$

$T_0 = 2.0 \text{ Seconds}$  - 2 db points  
 $T_n = 90 \text{ Seconds}$  .... 64 sec  
 .... 33 hz  
 $G_n = 12.653 \text{ N/A}$  zeta = 0.85  
 $C_p = 24.1 \text{ uF}$  displ: 37mv/m  
 $T_I = 80.4 \text{ seconds}$

$G = \frac{G_n}{M}$   $G \frac{\text{m}}{\text{sec}^2}$   
 $G = 25.3060$   
 $C = 2.4100 \cdot 10^{-5}$   
 $R_p = 5.6100 \cdot 10^5$   
 $R_I = 1.0700 \cdot 10^5$   
 $T_I = 80.4000$   
 $S_{00} := 2 \cdot \frac{\pi}{T_0}$   
 $S_{00} = 3.1416$   
 $\zeta = 0.8474$

Effective "natural Period" := (seconds) Damping :=  $\zeta := \frac{1}{2} \sqrt{\frac{1}{R_p} + \frac{(S_{00})^2}{rG}} \cdot \sqrt{\frac{T_I}{C} \frac{R_I}{C}}$

$T_n := 2 \cdot \pi \cdot \sqrt{C \cdot R_I \cdot T_I}$   $T_n = 90.4708$

Output function: Voltage per Acceleration: (assumes  $T_d \sim 0$ )

Generalized Equation:  $Y_i := \frac{1}{G \cdot C} \frac{s_i \cdot (1 + s_i \cdot C \cdot R_F)}{(s_i)^4 \cdot \left(\frac{R_F}{rG}\right) + (s_i)^3 \cdot \left(\frac{1}{rG \cdot C}\right) + (s_i)^2 \cdot s_i \cdot \left[\frac{(S_{00})^2}{rG \cdot C} + \frac{1}{C \cdot R_p}\right] + \frac{1}{C \cdot R_I \cdot T_I}}$

Setting Constants:

$k := \frac{1}{G \cdot C}$   $n := (C \cdot R_F)$   $a = \frac{R_F}{r \cdot G}$   $b := \frac{1}{r \cdot G \cdot C}$   $c := \frac{(S_{00})^2}{r \cdot G \cdot C} + \frac{1}{C \cdot R_p}$   $d := \frac{1}{C \cdot R_I \cdot T_I}$

Marking effective periods:  $F_1 := \frac{1}{T_I}$   $F_n := \frac{1}{T_n}$

Evaluating:  $A_i := k \cdot \frac{s_i \cdot (1 + s_i \cdot n)}{(s_i)^4 \cdot a + (s_i)^3 \cdot b + (s_i)^2 \cdot s_i \cdot c + d}$   $vel_i := Rc(A_i \cdot s_i)$   $maxv := \max(vel)$   $Ref. line: maxv = 1.6400 \cdot 10^3 \text{ Volt - sec / meter}$

