I. Feedback 101

The following discussion is highly non-rigorous. That having been said, it should be useful for solving many real-world feedback loop design problems.

Block diagrams are quite useful in analyzing feedback problems. Each block represents a signal transfer function and has an input and an output. In feedback loops the forward signal flow is most often drawn from left to right while feedback signals usually flow from right to left.

Consider the feedback loop in which a function of the output is fed back and subtracted from the input at a summing-point, creating an error term e(s). Most complex block diagrams can be reduced to this form for analysis. Note that sometimes the summing-point feedback polarity is shown as "+" in which case the required negative sign is included in the B(s) function.

We must assume that the forward elements making up A(s) and the feedback elements making up B(s) are sufficiently linear (at least within the expected range of operation), and that they can be represented by transfer functions which are polynomials of the complex variable "s", where s is related to frequency.

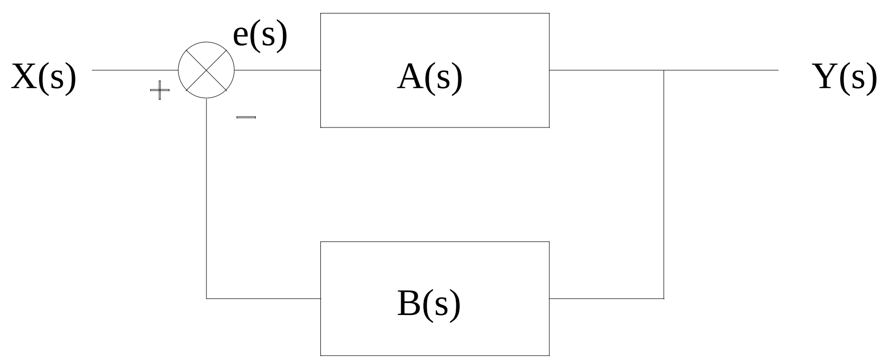

Figure 1 - Basic Loop

We can see that

$$Y(s) = e(s)A(s) = (X(s) - Y(s)\,B(s))\,A(s)$$

or

$$Y(s) = X(s)A(s) - Y(s)A(s)B(s)$$

giving

(1) $$Y(s)/X(s) = A(s) / (1 + A(s)B(s))$$

This is an expression for the ***closed-loop transfer function,*** sometimes called ***closed-loop gain***. This is the function we are attempting to achieve with the design.

The important term A(s)B(s) is called the ***loop gain*** or occasionally the ***open loop gain***.

Now consider what happens if A(s)B(s) >> 1   In the denominator of (1) the 1 becomes insignificant and is ignored, so that:

$$Y(s) / X(s) = A(s) / (A(s)B(s)) \quad \text{approximately.}$$

or

(2) $$Y(s) / X(s) = 1 / B(s) \quad \text{approximately}$$

At frequencies where the loop gain is large, the closed loop transfer function depends mostly on B(s).

Usually, the components in the B path, the feedback elements, are chosen for their accuracy, low-noise, predictability and stability, thus ensuring that the closed-loop transfer function is well characterized, under the conditions where A(s)B(s) is large. Under those conditions, the effect of the forward elements A(s) on the closed loop transfer function is minimal.

The loop gain is usually designed to be as high as possible over as wide a frequency range as possible. The practical limit on loop gain comes from the tendency of the system to self-oscillate with larger values.

Conditions on loop gain for stability (non-oscillation):

Physical considerations dictate that as frequency is increased, a frequency will eventually be reached where the magnitude of the loop gain A(s)B(s) has fallen to unity. That frequency is called the ***gain-crossover frequency***. If, at the gain-crossover frequency, the phase shift in A(s)B(s) has lagged by 180 deg (or more) from the initial 180 deg. associated with the negative polarity of the feedback, the system will oscillate.

The design goal is to shape the gain fall-off of the magnitude of the loop gain A(s)B(s) as frequency increases, in such a way that the phase lag in A(s)B(s) at gain-crossover is a comfortable amount less (say 20 to 50 deg less) than 180 deg. This difference is called the ***phase margin***. An adequate phase margin will help assure that (1) the closed-loop transfer function will not have excessive peaking near the gain-crossover frequency, and that (2) with normal system parameter variations, the loop will not oscillate.