Using the Excel Poles&Zeros workbook

October 25, 2015

Note: When opening the workbook 'Poles&Zeros.xls' if you are using the default security settings, you will be asked whether to Enable Macros. You should enable them. The workbook contains one macro which changes <ctrl>v so that it pastes only cell values, but not their formulas or formats. This can be particularly useful when copying and pasting pole and zero values from one place to another. The workbook was created using Excel 2002, so it should work properly with older versions. Newer versions of Excel should convert the file to the '.xlsx' format they use.

Poles and Zeros represent points on a plane (the S-plane) and so have two dimensions. The X, horizontal coordinate is called the real part and the Y, vertical coordinate is called the imaginary part. In general, the frequency response of any instrument may be described by a mathematical (polynomial) expression, with its numerator being a product derived from all the instrument's zeros, divided by an expression which is a product derived from all its poles.

For our purposes all poles and zeros will have a negative real part. They always appear in one of two forms, the first being single (Real) poles and zeros, which have an imaginary coordinate of zero, and which correspond to electrical R-C and R-L circuits or simple damped masses. Or they appear as second-order pairs which describe resonant systems, such as R-L-C circuits, pendulums or damped spring-mass systems. For under-damped systems, which will have a damping factor, $\zeta < 1$, the poles and zeros occur in complex conjugate pairs, both having the same negative value for the real part, with their imaginary coordinates being ± some value, or for over-damped systems, as a pair of real poles or zeros, both having imaginary parts = 0.

An instrument response may be plotted as log amplitude vs log frequency (Bode plot[1]), such as the one on page 3. The plot can be broken up into sections, each having approximately constant slope, which appear as relatively straight lines, called its asymptotes. A slope of +1 represents a region in which the amplitude is increasing by a factor of 10, 20dB or 1 decade, for each decade of increasing frequency. A slope of +2 increases by a factor of 100, 40dB or 2 decades, per frequency decade, while a slope of 0 is constant with frequency.

In frequency-response plots, each pole or zero defines a frequency where the slope of the plot changes, with a single pole causing the slope to decrease by 1 unit and a zero causing it to increase by 1. Poles and zeros will be at frequencies where the asymptotes intersect. Second-order pairs of poles cause the slope to decrease by 2 units, and depending on the associated damping factor, can create a curve which has a resonant peak. A second-order pair of zeros causes the slope to increase by two units and may exhibit a notch in the response. ***Zeros at zero frequency define the initial slope of the graph*** at the lowest frequencies, with one zero at zero giving an initial slope of +1, two zeros giving +2, etc.

The frequency response of an instrument is completely defined by a list of all its poles and zeros, along with a single additional number, which is required to pin down its mid-band sensitivity.

The worksheet 'Poles&Zeros' provides four tools for working with poles and zeros:

(1) allows a list of up to 9 poles and 5 zeros to be entered in the yellow cells, and from those it creates a table of the corresponding response amplitude vs frequency for that set, plotting a graph on chart 'Amplitude'. As is commonly done, the poles and zeros have their frequencies specified in radians per second rather than Hz. Note that the order in which the poles or zeros are listed doesn't matter. Of mostly academic interest, the chart 'S-Plane' plots their two-dimensional locations.

(2) computes the inverse of the net magnitude value of the entire pole-zero constellation at a selected mid-band frequency, F_n. This provides the normalization multiplier, A_n, required to obtain an amplitude response of 1.0 at that frequency. A chart 'Norm. Amplitude' shows the response curve, after normalization to an amplitude of 1 at frequency F_n.

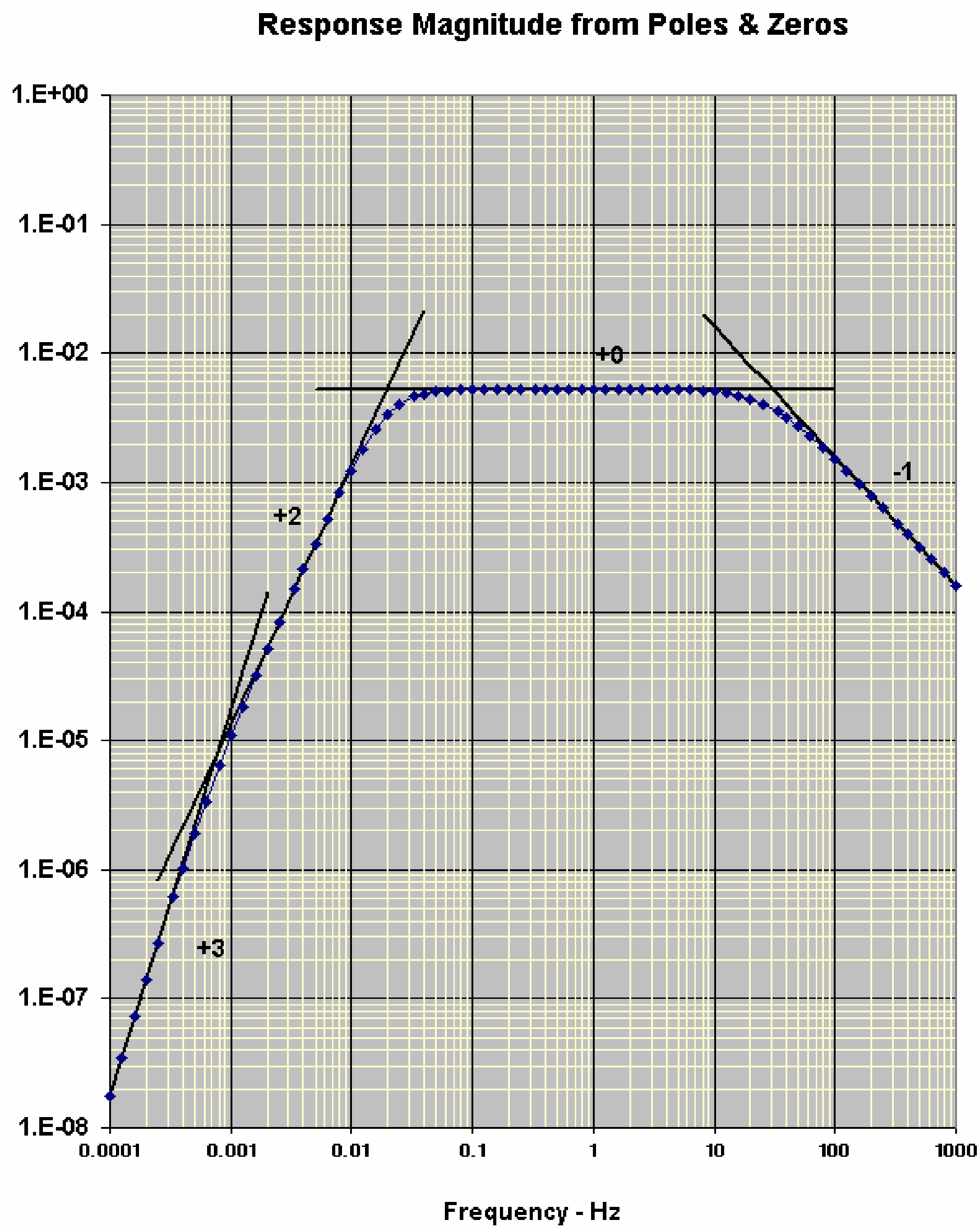
(3) computes the counts/Volt, Volts/count and max. counts, for a digitizer, given its maximum input voltage and number of bits. To be precise, the maximum count in the positive direction is one less.

(4) allows pole and zero components to be computed, by entering either their "corner" frequency or its period, in one of the yellow cells, or by entering one of those plus a damping factor for second-order pole or zero pairs. It also solves the inverse problem, computing frequency and period in terms of the pole or zero component values, additionally providing the damping factor value for second-order pairs. Note that all valid second-order pairs must either have both their imaginary components = 0 (over damped) or their real parts must be exactly equal and their imaginary parts exactly equal but of opposite signs. (under damped).

Finally, the worksheets, 'Instruments', are a place for archiving the poles, zeros and gain factors for specific instruments, where is also computed the 'CONSTANT' normalization factor associated with each instrument, as required in the 'Response' file, used by a noise-modeling program.

The worksheets and charts are protected, so that only the yellow cells may be altered. This prevents accidentally changing the formulas contained in the other cells. If protected data must be altered, you can select 'Tools' > 'Protection' > 'Unprotect Sheet' which will allow all cells to be edited. If making changes, it may be a good idea to keep a copy of the original workbook, just in case some formula gets changed by accident.

Note that the values of cells and ranges of cells may be copied and pasted into yellow cells rather than having to type them. When pasting, to avoid altering the format of the recipient cell(s), use <ctrl>v (as modified in this workbook), or right click on the cell, and select 'Paste Special' > 'Values' > OK to paste only cell values. Only the copied value will be pasted, not the cell formulas or colors.



This is a frequency plot of the net magnitude associated with the poles and zeros which describe the response of a particular broadband vertical seismometer, here, Napa#1 with an added preamp.

In this example there are three zeros at 0 frequency which gives the response curve an initial slope of +3 decades per frequency decade. A single pole in the preamp at 0.00077 Hz (1300 seconds) reduces the slope to +2. A conjugate pair of poles at about 50.1 seconds (0.02 Hz), reduces the slope by 2, to zero, creating the flat portion of the instrument's velocity response. Another single pole at 30Hz starts its response falling with -1 slope.

It may be convenient to change the data so that their mid-band amplitude value is 1.0. To do that, in section (2) of worksheet 'Poles&Zeros' the constant, A_n is computed for some frequency, F_n, chosen to be in the middle of the flat part of the response. We see that for F_n = 1.0 Hz, A_n = 188.6..... and by multiplying all the data points by that number, the flat part of the resulting curve will have a magnitude close to 1.0 .

An exercise in using Poles and Zeros.

Let's look at how we might construct the velocity response of a simple broadband instrument.

Assume we know that its response has a low-frequency corner at 20 seconds, falling with a slope of 2 below that, is flat between the low and high corners and that it has a high frequency corner at 15Hz, above which it falls with a slope of -1.

You will be going back and forth between the P&Z tab, where you will calculate and enter poles and zeros, and the Amplitude tab where you can look at the computed response magnitude curve.

First, let's clear the deck. In section (1) of worksheet 'P&Z', in the yellow cells under 'Zeros - Rad./Sec' and 'Poles - Rad./Sec." clear all the values. It will help you if you know how to select blocks of cells in Excel. To clear an area quickly, first select that block and then use the 'Delete' key.

Now go to chart 'Amplitude'. With no poles or zeros the response function magnitude is simply constant with a value of 1.

To put in a single zero at zero frequency, enter '0' in the both the Real and Imaginary columns of the first row of yellow cells in the 'Zeros' section. Now we can see that the response amplitude is a line rising at a rate of +1 decade per frequency decade. Note that it has the value 1 at a frequency of $1/2\pi$ (=0.15915....Hz), which has a period of 2π seconds. In general, the spreadsheet does its computations using angular frequency ω, radians per second, where frequency in radians per second = 2π * frequency in Hz. So our response has a value of 1 at $1/2\pi$ Hz which is a frequency of 1 radian per second.

Add a second zero at zero. Type '0' in the two yellow cells in the second row of the 'zeros' section. Now the amplitude rises at +2 decades per frequency decade, again equal to 1 at $1/2\pi$ Hz.

To make our low frequency corner requires adding a second-order conjugate pair of poles to flatten the curve at 20 seconds. This represents a spring-mass-like response and we will set its damping factor to 0.7, close to the "ideal" value of $1/\sqrt{2}$. But first we need to compute the real and imaginary parts for the poles we need. In section (4) 'Pole-Zero Calculators, under 'Second-Order Pole-Zero Pairs. In the yellow cell for T_0-Sec. enter 20, and in the yellow cell to its right, enter **.**7 for the damping. Note that if we knew the corner frequency rather than its period, we could use the calculator immediately below. In the four cells in two rows of the 'Real' and 'Imaginary' columns you should find the real values of -0.2199... and imaginary values of ± 0.2243....

Now to enter the pole-pair into section (1), select the four cells containing the values we just calculated and press <ctrl>c to copy them. Or you can use 'Edit' > 'Copy' or right-click and 'Copy'. Then click on the first empty yellow cell in the 'Real' column of the 'Poles - Rad./Sec.' section of (1) and press <ctrl>v to paste their values into four yellow cells. Note that in this workbook <ctrl>v has been modified so that it pastes only the values of the cells which were copied, not their formulas, cell colors or borders. If you prefer you could paste values by using 'Edit' > 'Paste Special' > 'Values' > OK or you could right-click on the target cell, then select 'Paste Special' > 'Values' > OK.

If you got the 4 values of the pole-pair entered successfully, you should see that the curve on the amplitude chart has been flattened at 0.05Hz (20 seconds) to a value of 1.

Now for the high-end corner, we will use the Pole-Zero Calculator to compute a pole for a single-pole high-frequency corner at 15 Hz. In the section for 'Single (Real) Pole or Zero, in the yellow cell in the 'f_0 - Hz' column, enter 15. Note that if we knew period, rather than frequency, we could use the calculator in the row below. Now select and copy the two cells it computed in the 'Real' and 'Imaginary' columns (the imaginary cell should be 0). Go to section (1) and select the first empty yellow cell in 'Real' column of the 'Poles' section and use <ctrl>v to paste in the real and imaginary parts of the computed pole.

The amplitude response should now fall with a -1 slope with a corner frequency of 15 Hz and a mid-band amplitude around 0.0106.

If you wanted to go on to use the data from these poles and zeros to describe the response of a real instrument you would have to scale its amplitude properly. First you should scale the collection of poles and zeros to have an amplitude of 1 in the flat portion of the curve. To do that, go section (2) on the 'Poles&Zeros' worksheet, and enter in the 'F_0 - Hz' column a frequency value which is somewhere in the middle of the flat portion of the amplitude curve. In this example 1Hz looks good. If the response curve obtained from the poles and zeros is multiplied by A_n , in this example x 94.452..., the resulting amplitude curve 'Norm. Amplitude' will have a value of approximately 1 throughout its flat part.

To get the entire instrument response, you would need to multiply A_n by the instrument's mid-band sensitivity or generator constant, S_v. Assume that it is 1500 Volts per meter/sec. And if you wanted the curve to be in counts per meter/second, you could use section (3) for the kind of digitizer you were using. Enter its maximum input voltage and number of bits to compute its counts/Volt and then multiply the previous result in Volts per m/s by that, giving you the curve in counts per m/s. The three scaling numbers we found, A_n , S_v and the digitizer counts/V could all be multiplied together to calculate a single normalizing factor to get from poles and zeros to the instrument response in counts/m/s. In this example, if you were using a 10V, 16-bit A/D, that factor would be 94.453 * 1500 * 3276.8 = 4.643E+8 counts per m/s

Now suppose you wanted to see what the instrument response looks like in terms of Acceleration, m/s^2 instead of Velocity, m/s . They say that acceleration is the time derivative of velocity, so to get acceleration you need to differentiate something, somehow, with respect to time. Using poles and zeros to describe the response makes that trivial, no calculus required. To see the response with respect to an acceleration input, simply delete one of the zeros at zero, one of the 0,0 pairs. To get the response with respect to displacement, you can see the result of integrating, by just adding another 0,0 pair. Sometimes calculus isn't so hard.

The 'Amplitude' charts attempt to set their axis ranges to match the data, but they don't always adjust themselves well enough. If you needed to fix that, select 'Tools' > 'Protection' > 'Unprotect Sheet' , then right-click each axis in turn and select 'Format Axis' > 'Scale' tab and change the Minimum and Maximum values of that axis to something you like better.

This kind of pole-zero data is used in dataless SEED file headers. IRIS has a program, PDCC which helps create those[2]. In its user manual you can see how really complicated it can get when you try to create a format which will accurately describe all conceivable combinations of instruments, amplifiers, digital filters and digitizers. However, the poles and zeros, normalizations and instrument sensitivities are essentially what you have already been using here, just with many more options.

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[1] http://lpsa.swarthmore.edu/Bode/Bode.html

[2] http://www.iris.edu/pub/programs/pdcc/PDCC_3.8_User_Manual.pdf