# Parameter Scaling in Vertical Feedback Seismomenters

When analyzing feedback seismometer performance in the Excel workbook 'Loop', the assumption is made that the Forcing Coil, the Position Sensor, the Spring and the Center of Mass are all located at a single point along the boom. This is not generally true and for the spreadsheet, the measured parameter values must be corrected so that they will appear to be acting at the assumed point. In the spreadsheet, these corrected values are characterized as 'effective' values.

Additionally, the workbook incorrectly computes the boom's moment of inertia about its pivot, by assuming that its mass is concentrated at a single point located at the center of mass. The boom is in fact a distributed structure, having a moment of inertia about its pivot which is greater than that of the point mass. In order to have everything acting at a single point, two mass values must be used. The first is the actual mass, acting at the center of mass, corrected to locate it at the common reference point, and used in certain of the calculations. The second is a larger mass which results in the proper rotational moment of inertia when located at the common point and which is used in other calculations as appropriate. Currently there is no mechanism for including this second mass value in the spread sheet, though that is in the process of being corrected.

Referring to Figures 1 and 2, the variables which we will use are related as follows:

1) $M_0 = M\, d_1/d$ kg

2) $M_1 = M\, (d_3/d)^2$ kg

3) $r_t = r\, d/d_4$ V/m

4) $G_n = G\, d_2/d$ N/A

5) $K_0 = K(d_5/d)^2$ N/m

(Note that the spreadsheet derives $K_0$ from the measured spring-mass period and it is not required to be entered as an input parameter.) Taking $K_0$ from the spreadsheet, we can compute the 'measured' spring constant of the real spring, located at distance $d_5$ as

6) $K = K_0(d/d_5)^2$ N/m

Here, $M_0$, $M_1$, $r_t$, $G_n$ and $K_0$ are the 'effective' values used in the spreadsheet, and M, r, G and K are the 'true' values, as would be measured at the actual locations of the various elements. Note that the value of 'd' is arbitrary and any convenient value may be used. It would probably be simplest to make $d = d_1$, the distance to the center of mass, or perhaps $d = d_2$ the distance to the forcing coil axis, just so long as you use the same value for 'd' in all calculations.

**M** is the measured mass of the boom assembly, located at the center of mass, which is at horizontal distance $d_1$ from the pivot.

**r** is the sensitivity of the position sensor (with its amplifiers) as measured at its center, located at radial distance $d_4$ from the pivot.

**K** is the incremental force constant of the spring, located at a radial distance $d_5$ from the pivot.

**G** is the force constant of the feedback coil, located at a radial distance $d_2$ from the pivot.

Distances $d_2$, $d_4$ and $d_5$ are physical dimensions which may be measured. The location of the center of mass, $d_1$ and the radius of inertia, $d_3$ will require some experimentation to determine. $d_1$ may be determined by hanging the boom assembly from a convenient point and estimating where a vertical line through the support would intersect the plane of symmetry of the boom, taking $d_1$ as the distance from the pivot to the intersection point.

To determine $d_3$ requires somewhat more work. One accurate method is to incorporate the boom into a torsion pendulum and measure its period. A suitable length of music wire can be used for the torsion spring, suspending the boom on its side from a point chosen so that the hinge axis is exactly vertical. After the oscillation period of the boom, $T_b$ is measured, the boom is replaced by a circular disk of known mass supported horizontally from its center, having mass $M_d$ and radius $R_d$, and its period of oscillation $T_d$ is measured. Knowing that the period of oscillation of a torsion pendulum, in general, is given by:

7) $T=2\pi\sqrt{(I/K)}$ sec

where I is the moment of inertia of the suspended object and K is the torsion constant of the suspension wire, it can be determined that the moment of inertia of the boom about its center of mass, when suspended from the same wire as the disk is,

8) $I_{bc} = (M_d R_d^2/2)(T_b/T_d)^2$

Now, having determined the values of $I_{bc}$ and $d_1$, we can weigh the boom to determine its mass, M, and then make use of the parallel axis theorem to solve for the moment of inertia of the boom about its pivot axis,

9) $I_{bp} = M\, d_1^2 + I_{bc}$

This makes the radius of inertia of the boom about its pivot,

10) $d_3 = \sqrt{(d_1^2 + I_{bc}/M)}$

which is the value we were seeking.

Then from 2) we have that $M_1 = M\,(d_3/d)^2 = M\,(d_1^2 + I_{bc}/M) / d^2$

which allows us to calculate $M_1$ as a function of the rotational moment of inertia which was measured.

11) $M_1 = M\, d_1^2/d^2 + I_{bc}/d^2$

## Real World

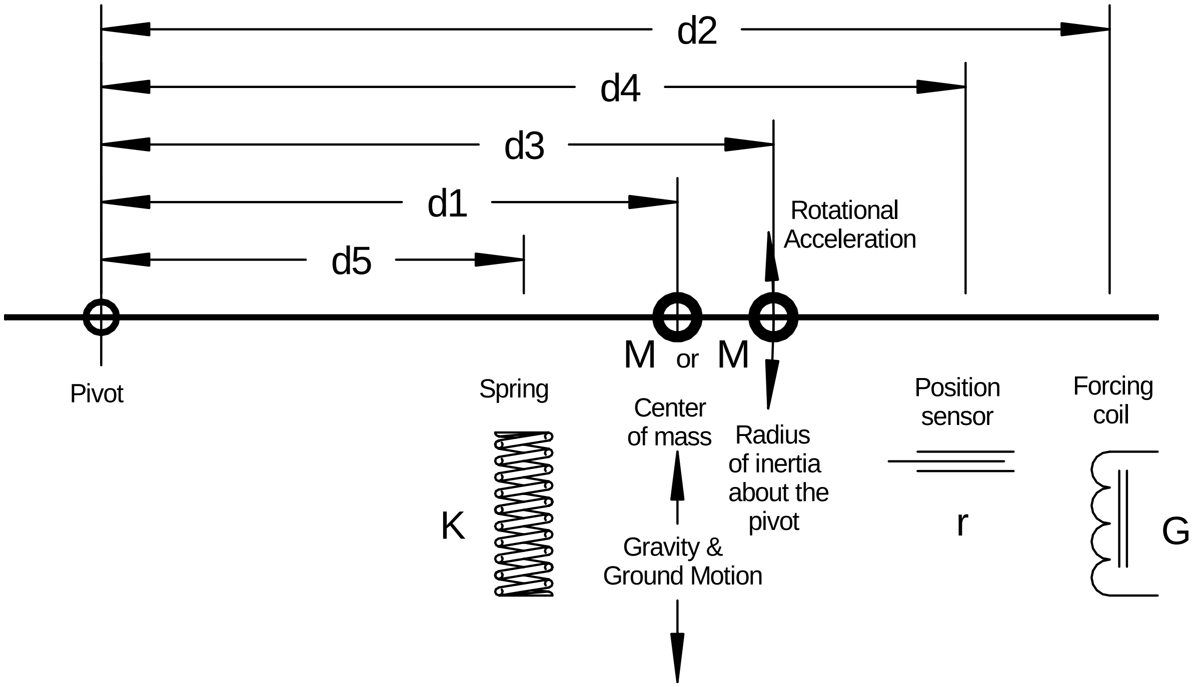



**Figure 1**

## Point-Mass Model

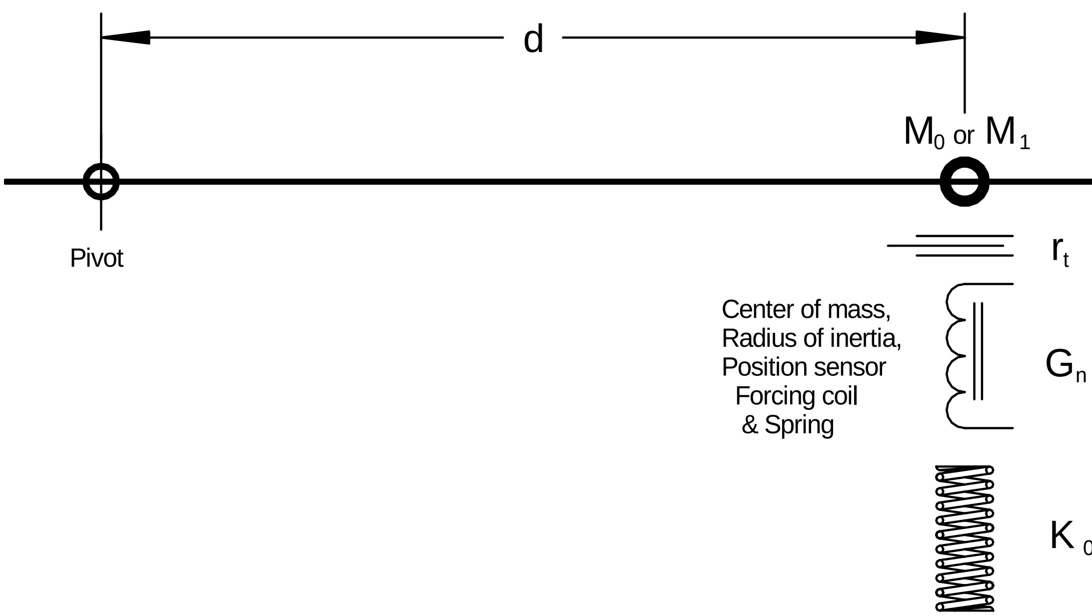



**Figure 2**

## Conversions between linear and rotary motion of the boom.

These all assume that the boom rotation angle θ, is sufficiently small that the approximation, $\sin\theta = \theta$ radians, is valid. Here 'd' is the distance from the pivot to the point of measurement.

12) $dz = d \cdot d\theta$ Rotation $d\theta$ to vertical displacement dz

13) $V = dz/dt = r\, d\theta/dt = d \cdot \omega_p$ Rotational rate $\omega_p$ to vertical velocity V

14) $A = d^2z/dt^2 = r\, d^2\theta/dt^2 = d \cdot a$ Rotational acceleration a to vertical acceleration A

15) $d\theta = dz/d$ Vertical displacement d to rotation $d\theta$

16) $\omega_p = d\theta/dt = 1/r\, dz/dt = V/d$ Vertical velocity d to rotational rate $\omega_p$

17) $a = d^2\theta/dt^2 = 1/r\, d^2z/dt^2 = A/d$ Vertical acceleration A to rotational acceleration a

Note that $\omega_{bp}$ is the instantaneous rate of rotation of the boom about its pivot (in radians per second), as distinct from plain ω, the frequency of an assumed sinusoidal ground motion, also expressed in radians per second.

# Equations of Motion

### Ground motion acting on the boom

Given that 'F' is the acceleration force on the center of mass due to ground motion.

18) $F = MA_g$ , where $A_g$ is the acceleration associated with ground motion.

Torque, 'Q', acting on the boom assembly due to ground motion

19) $Q = F\, d_1 = M\, d_1\, A_g = M_0\, d\, A_g$

Rotational Acceleration, 'a', of the boom, due to ground motion

20) $a = Q / I_{bp} = M_0\, d\, A_g / I_{bp} = A_g\, d\, M_0 / (M_1\, d^2) = A_g/d\;\; M_0/M_1$

Linear acceleration of the point at distance 'd' from the pivot

21) $A_d = d\, a = A_g\, M_0/M_1 = A_g\, d\, d_1/d_3^2 = A_g\, d/(d_1 + I_{bc}/Md_1)$

So the acceleration of point 'd' is decreased by the factor $M_0/M_1$ from the ground acceleration, where $M_0/M_1$ also $= d\, d_1/d_3^2 = d/(d_1+I_{bc}/Md_1)$

Note that in the STS-2 block diagram, in which 'd' is chosen $= d_2$, (which they also assume $= d_4$), they show this factor as $d_1d_2/d_3^2$ which agrees with 21) when 'd' is replaced by '$d_2$'.

**Feedback coil force acting on the boom**

Assume that the feedback coil exerts a force, 'F', on the boom at distance '$d_2$' from the pivot. The torque 'Q' applied by the feedback coil to the boom is

22) $Q = F\, d_2$ where F is the feedback force.

The angular acceleration, a, due to the torque 'Q'

23) $a = Q/I_{bp} = F\, d_2/I_{bp}$

From 14) this results in a vertical acceleration, 'A', at radius 'd', of

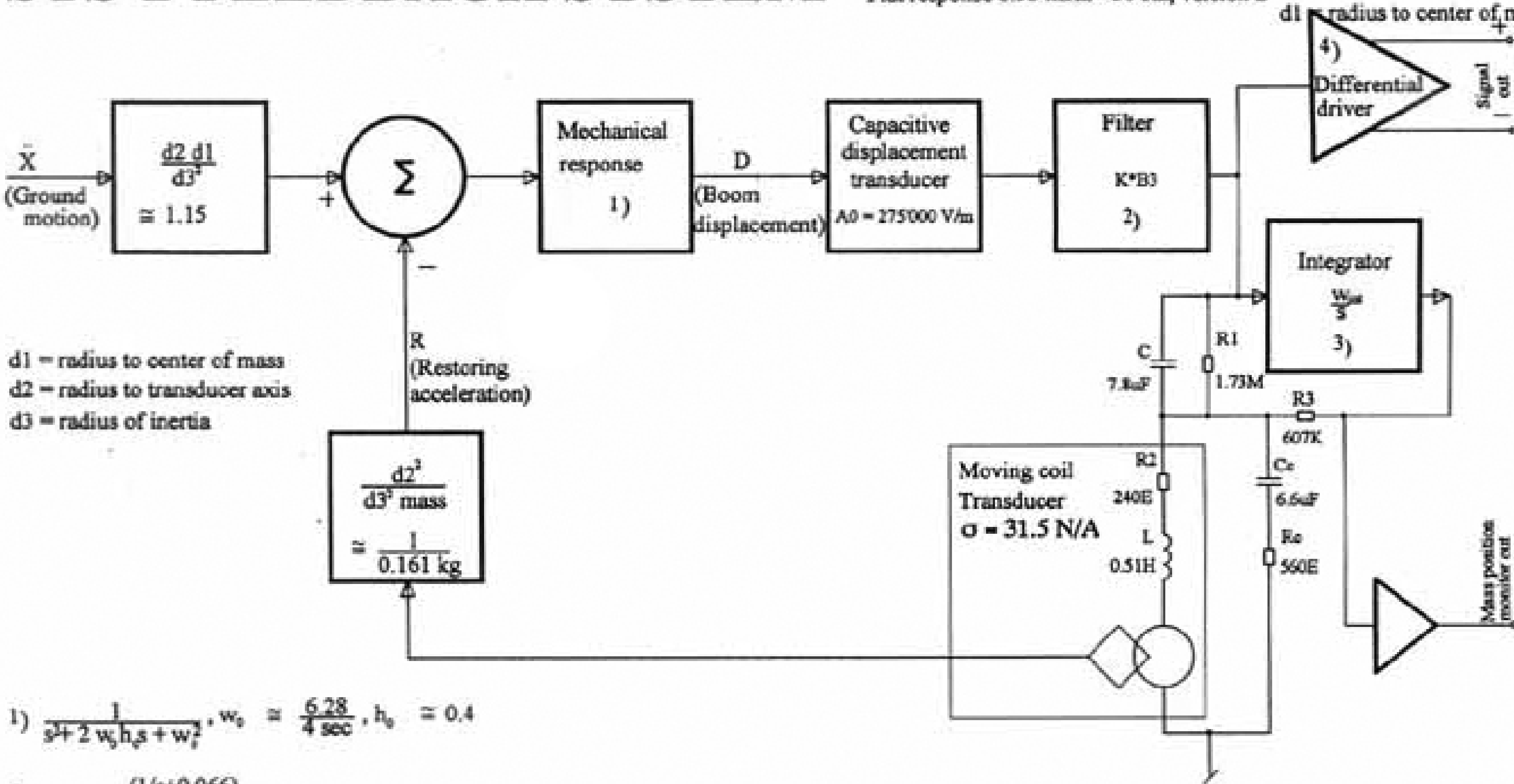
24) $A = a\, d = F/M_1\ \ d_2/d\ = Fdd_2/\ Md_3^2$

In the STS-2 block diagram it is assumed that d = d2 so the acceleration 'A' of the point at distance 'd2' due to a feedback coil force of 'F'

25) $A= F\, d_2^2/Md_3^2$ which agrees with the STS-2 block diagram.

# STS-2 FEEDBACK SYSTEM

Flat response 8.33 mHz - 50 Hz, version 2




1) $\frac{1}{s^2+2w_0h_0s+w_0^2}$, $w_0 \cong \frac{6.28}{4\,sec}$, $h_0 \cong 0.4$

2) $K = \frac{(1/s+0.066)}{(1/(475s)+0.066)}$ (Inverse filter)

$B3 = \frac{1}{(1+7.52e\text{-}5s)(1+9.94e\text{-}5s+4.72e\text{-}9s^2)}$ (Bessel 3rd order 1600 Hz)

3) $w_{int} = \frac{1}{77.1\,sec}$

4) With low-pass filter: $\frac{1}{s/w_{dd}+1}$, $w_{dd} = 6.28 \cdot 40\,/sec$