

The Scientist and Engineer's Guide to  
**Digital Signal Processing**

Second Edition

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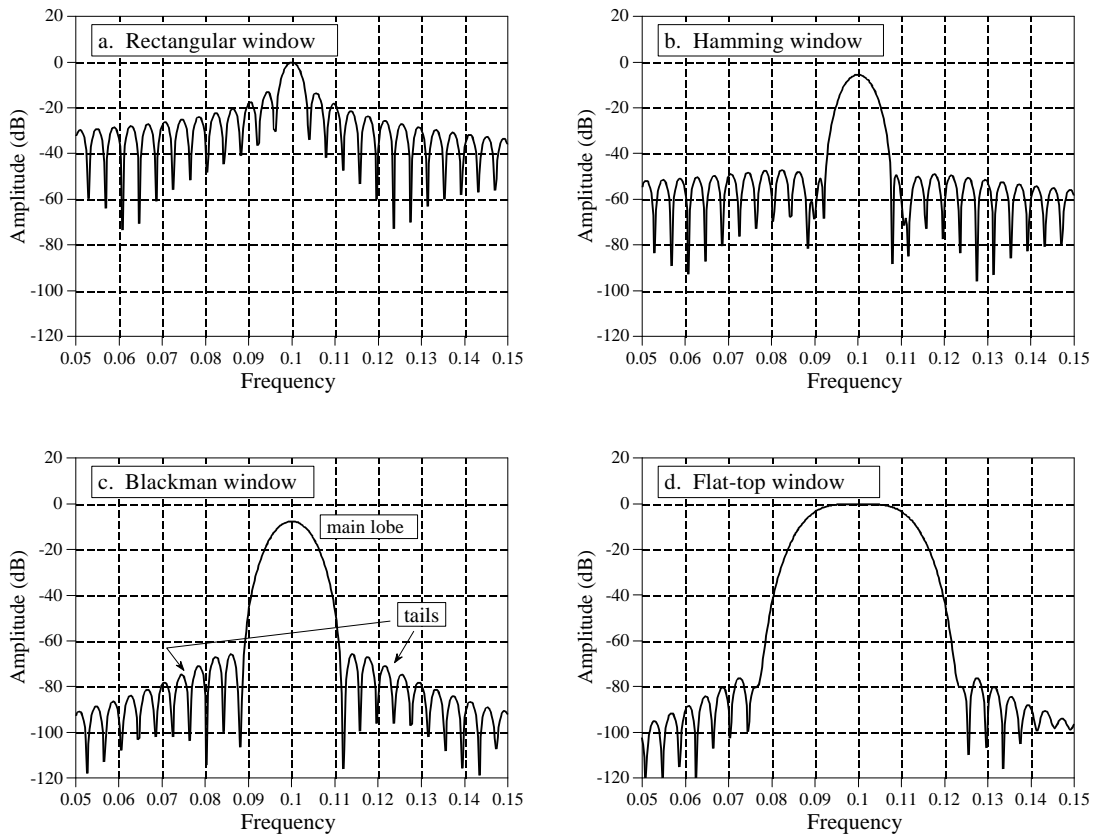


FIGURE 9-5

Detailed view of a spectral peak using various windows. Each peak in the frequency spectrum is a central lobe surrounded by tails formed from side lobes. By changing the window shape, the amplitude of the side lobes can be reduced at the expense of making the main lobe wider. The rectangular window, (a), has the narrowest main lobe but the largest amplitude side lobes. The Hamming window, (b), and the Blackman window, (c), have lower amplitude side lobes at the expense of a wider main lobe. The flat-top window, (d), is used when the amplitude of a peak must be accurately measured. These curves are for 255 point windows; longer windows produce proportionately narrower peaks.

of *sampling* the frequency spectrum's continuous curve. For example, if  $N$  is chosen to be 1024, the spectrum's continuous curve will be sampled 513 times between 0 and 0.5. If  $N$  is chosen to be much larger than the window length, the samples in the frequency domain will be close enough that the peaks and valleys of the continuous curve will be preserved in the new spectrum. If  $N$  is made the same as the window length, the fewer number of samples in the spectrum results in the regular pattern of peaks and valleys turning into irregular tails, depending on where the samples happen to fall. This explains why the two peaks in Fig. 9-4a do not look alike. Each peak in Fig 9-4a is a *sampling* of the underlying curve in Fig. 9-5a. The presence or absence of the tails depends on where the samples are taken in relation to the peaks and valleys. If the sine wave exactly matches a basis function, the samples occur exactly at the valleys, eliminating the tails. If the sine wave is between two basis functions, the samples occur somewhere along the peaks and valleys, resulting in various patterns of tails.

## Moving Average Filters

The moving average is the most common filter in DSP, mainly because it is the easiest digital filter to understand and use. In spite of its simplicity, the moving average filter is *optimal* for a common task: reducing random noise while retaining a sharp step response. This makes it the premier filter for time domain encoded signals. However, the moving average is the *worst* filter for frequency domain encoded signals, with little ability to separate one band of frequencies from another. Relatives of the moving average filter include the Gaussian, Blackman, and multiple-pass moving average. These have slightly better performance in the frequency domain, at the expense of increased computation time.

### Implementation by Convolution

As the name implies, the moving average filter operates by averaging a number of points from the input signal to produce each point in the output signal. In equation form, this is written:

#### EQUATION 15-1

Equation of the moving average filter. In this equation,  $x[ ]$  is the input signal,  $y[ ]$  is the output signal, and  $M$  is the number of points used in the moving average. This equation only uses points on *one side* of the output sample being calculated.

$$y[i] = \frac{1}{M} \sum_{j=0}^{M-1} x[i+j]$$

Where  $x[ ]$  is the input signal,  $y[ ]$  is the output signal, and  $M$  is the number of points in the average. For example, in a 5 point moving average filter, point 80 in the output signal is given by:

$$y[80] = \frac{x[80] + x[81] + x[82] + x[83] + x[84]}{5}$$