Astatic Geometries

In the FBV designs, the boom and leaf spring represent an astatic spring-mass system, as originally proposed by Wielandt and Streckeisen[1]. This is an elegant and effective solution to supporting a boom in such a way that it may be easily incorporated into the design of a force-balance feedback vertical seismometer. However it is sufficiently elegant that it is difficult to understand what is going on mechanically in such an instrument. In principle, their design creates an astatic configuration which, with careful adjustment of the spring, results in a vertical pendulum having a natural oscillation frequency which is quite low.

Why is a long free period good? For a spring-mass assembly its natural period is given by $\sqrt{(M/K)}$, where M is its mass and K is the effective spring constant of the spring, its net force/displacement (or torque/rotation). For a given mass, a long natural period implies a small value for K, and the smaller you make K, the more easily the feedback loop can control the motion of the boom at longer periods, so the instrument will perform slightly better. It is worth noting that some instrument designs, such as those made by Guralp, use relatively short free periods, around ½ second. But, in order for them to work as well as ones with longer free periods, the position sensor in their feedback loop must be made correspondingly more sensitive. However, if you used the more sensitive Guralp sensor in an instrument having a longer free period, it would perform even better.

The reason that astatic geometries are used is that they allow a spring having a relatively high K to work with gravity to appear as though its K were much lower.

To understand the general astatic principle, consider the pendulum in Figure 1, in which a coil spring has been attached from a point on the pendulum to a fixed point above its pivot. Then as the pendulum is displaced to the side, gravity will try to restore the pendulum to its rest position. The further the pendulum is moved, the greater will be the restoring force (torque) from gravity. However as the pendulum is being displaced, the spring will begin to exert its own torque, attempting to move the boom away from the center, acting in opposition to the effect of gravity. One could imagine that some configuration might be found for the location of points A and B and for the spring strength, which, with a given mass, would nearly match the effect of gravity. Such a pendulum would oscillate with a period which is much longer than its length would suggest. Note that if the spring were made very slightly stronger, the pendulum would no longer seek the center rest position but would have stable positions at the extremes of its swing.

Although, this configuration doesn't actually work in practice, it is somewhat similar to the operating principle of the FBV astatic leaf spring, in which the the spring and gravity torques can be almost perfectly matched by adjusting the spring length (by a few thousandths of an inch). Small changes will have a dramatic effect on the free period.

[1]Wielandt, E. and G. Streckeisen (1982), The leaf-spring seismometer: Design and performance, *Bull. Seismol. Soc. Am*, **72**(6), 2349-2367

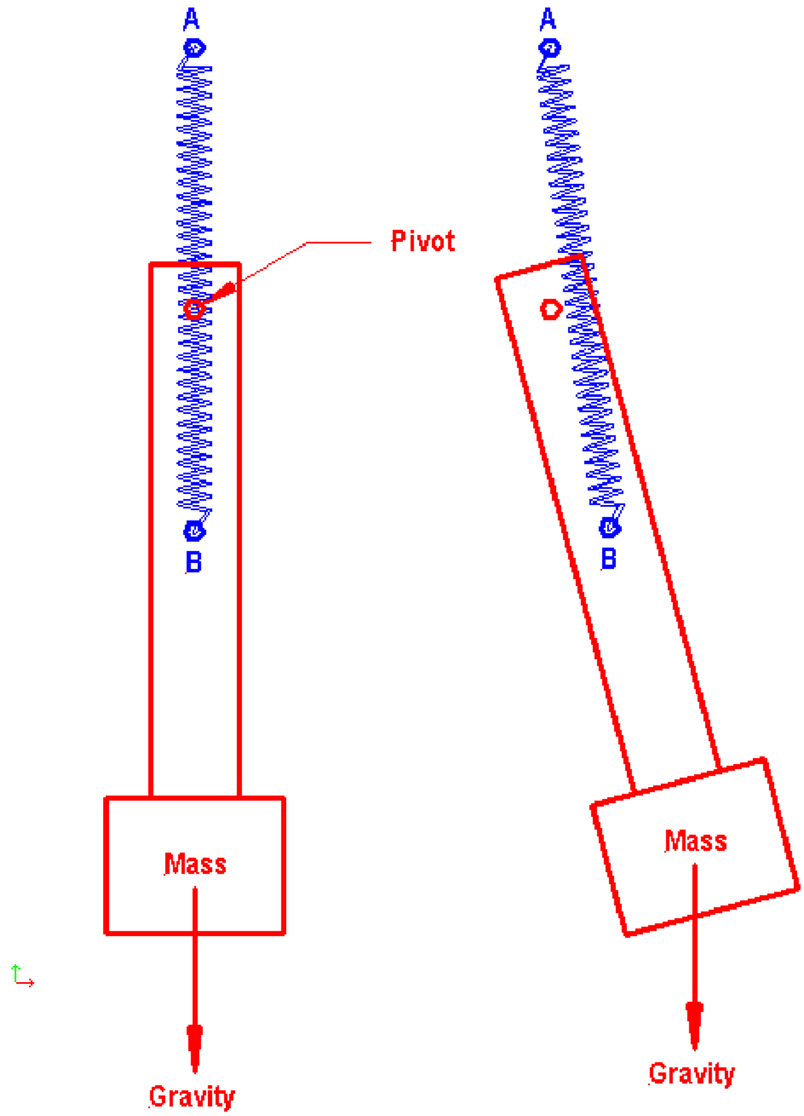

Figure 1

Why target 3 seconds for the FBV free period? A more accurate statement of the target period would be "about 3 seconds or longer". Perfection would be achieving an infinite period, in which the effective restoring torque constant K would approach zero. However, it is very difficult to determine when one has obtained an infinite period, and the more observable 3 seconds appeared to be adequately long for excellent performance.

With the feedback loop active, the instrument will operate properly even when the the spring is adjusted for a bi-stable configuration. In the spring modeling program that bi-stable situation is indicated by a negative period. Considering that, it would be more precise to suggest that the ***absolute value*** of the period should be around 3 seconds or greater. The only way I can think of to measure the value of a negative period would be to operate the instrument with the feedback active, plot its loop gain vs frequency and observe the location of the peak which occurs at the resonance of the spring-mass. Free periods of 3 seconds and -3 seconds should both show a peak at 1/3 Hz.