Analysis of the Yuma Vertical Sensor

The Mechanical System

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Introduction

The FBV, Force Balance Vertical, series of seismometers make use of the astatic leaf spring principle, conceived by Prof. Erhard Wielandt and Gunnar Streckeisen, which was described in their paper in 1982.[1] This is an elegant and effective solution to supporting the seismometer boom in such a way that it may easily be incorporated into the design of a force-balance feedback vertical sensor. However, it is sufficiently elegant that it is not so easy to understand what is going on mechanically. In principle, their design creates an astatic configuration which, with careful adjustment of the spring, results in a vertical pendulum having a natural oscillation period which can be of the order of several seconds or longer.

The leaf spring in the Yuma2, the current FBV design, applies a force and torque to the boom sufficient to support it in a level orientation. But because of its astatic geometry the spring exerts only a small restoring torque on the boom, allowing it to oscillate with a free period of several seconds, so that the Yuma's electronics can be firmly in control of the boom motion, particularly at the long-period end of its response.

The Astatic Principle

To understand the general astatic principle, consider the horizontal pendulum in Figure 1, in which a coil spring has been attached from a point on the pendulum to a fixed point above its pivot. Then as the pendulum is displaced to the side, gravity will try to restore the pendulum to its rest position. The further the pendulum is moved, the greater will be the restoring force (torque) from gravity. However as the pendulum is being displaced, the spring will begin to exert its own torque, attempting to move the boom away from the center, acting in opposition to the effect of gravity. One could imagine that some configuration might be found for the location of points A and B and for the spring strength, which, with a given mass, would nearly match the effect of gravity. To such a pendulum the gravity acceleration, 'g', would appear to be much less, so it would oscillate with a period which is much longer than its length would suggest. Note that if the spring were made very slightly stronger, the pendulum would no longer seek the center rest position but would have stable positions at both extremes of its swing.

Although, this configuration doesn't actually work in practice, it is somewhat similar to the operating principle of the FBV astatic leaf spring, in which the torques from the spring and from gravity can be matched almost perfectly by adjusting the spring length (by fractions of a mm).

[1] Wielandt, E. and G. Streckeisen (1982), The leaf-spring seismometer: Design and performance, *Bull. Seismol. Soc. Am*, **72**(6), 2349-2367

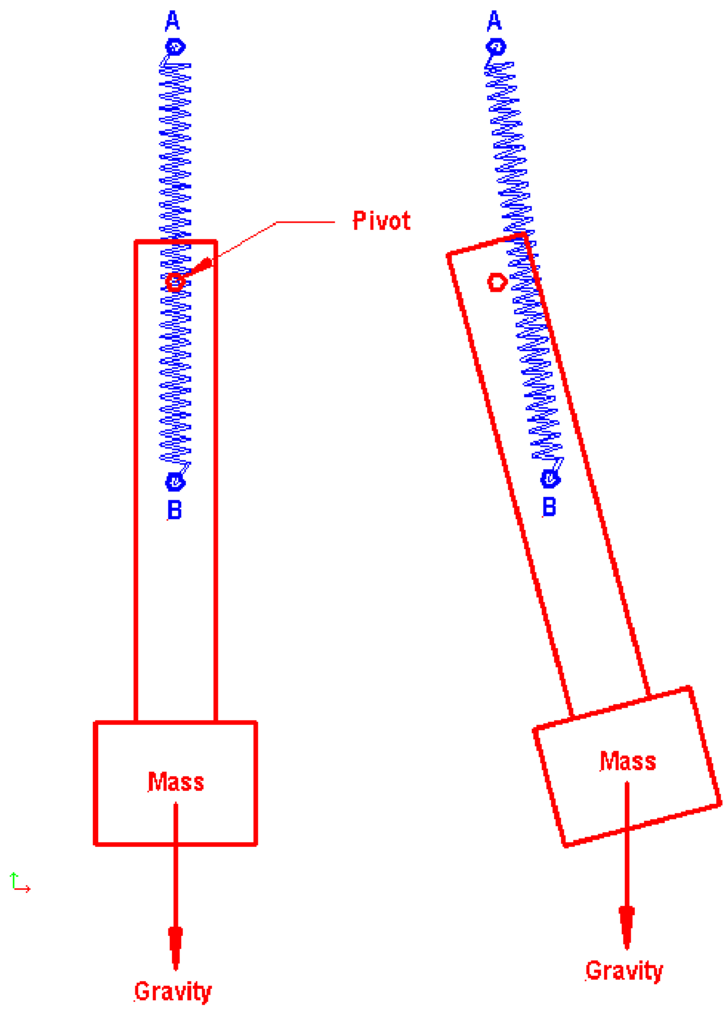


Figure 1
An astatic pendulum design (that doesn't work)

The FBV Geometry

The FBV astatic spring configuration has a leaf spring rigidly attached at one end to the base plate and at the other, to the boom. This spring configuration has three geometric degrees of freedom, which are the end force magnitude, F, force direction, ϕ, and spring end-moment Q_N; which will be related to the spring-end x coordinate X_N, y coordinate Y_N, and its degree of bending, $(\theta_N - \theta_0)$, as well as to the stiffness of the spring. In addition, the locations of the pivot, the spring-boom attachment and the center of mass, as well as the mass magnitude must be specified in order to completely define the astatic geometry, and arriving at the "best" or even an acceptable combination by experimentation alone is likely to be quite difficult. Note that, in order to have the sensitive axis precisely vertical, with no sensitivity to horizontal motion, Yc must = Yp.

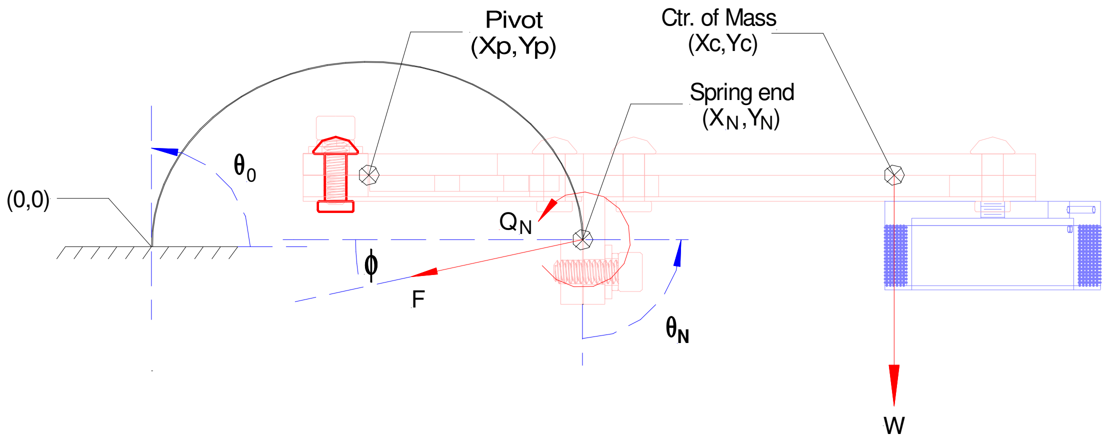


Figure 2

Analysis

To determine the boom's natural period, it is easiest to analyze it as a rotating spring-mass, much like the balance wheel of a mechanical alarm clock. One element of the system is a distributed mass rotating about an horizontal axis, consisting of the coil, all the boom elements and any trimming weights, which is described by its mass, center of mass location and rotational moment of inertia. Attached is the equivalent of a torsion spring. This is a "virtual" spring, arising from the sum of the moments acting on the boom, whose magnitude is made to be approximately proportional to angular deflection, and which acts in a direction tending to restore the boom to its horizontal rest position.

The virtual spring is the resultant of two major competing moments which both vary slightly with boom rotation, one from the gravity force W acting on the center of mass, and tending to rotate the boom downward (CW), the other from the leaf spring, acting upward (CCW). In addition, the pivot flexures add a small amount to the total moment. This virtual spring may be approximated by a rotational spring constant, defined as the, assumed constant, rate at which the net restoring moment changes (per radian) of boom rotation. Once we have determined the moment of inertia and effective spring constant, the free period of oscillation is easily computed.

In order to observe the nature of the restoring moment, we can compute the sum of the spring and gravity moments, and plot it vs. the up-down boom rotation angle γ. For a workable design, we want two conditions to be fulfilled. First, for equilibrium, the net restoring moment should be zero when the boom is in its horizontal rest position at $\gamma = 0$.

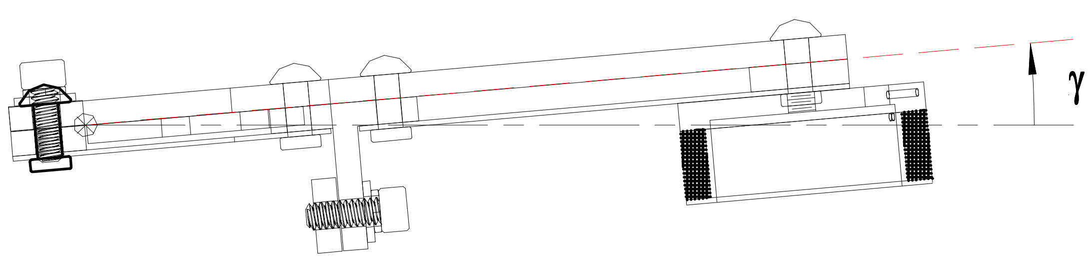


Figure 3

Second, in order to have a stable equilibrium, the upward moment should increase as the boom rotates downward, and then reverse and become a net downward moment as the boom is rotated above zero degrees. This is of significance because it is easy to find configurations where the latter conditions are reversed, in which case, in the absence of force feedback the boom will be bistable, moving to either its upper or lower limit stop when released. As plotted in the "Net Moment" chart, a stable net moment curve will intersect the Q = 0 axis with a negative slope, i.e. from upper left to lower right. It would also be desirable for the net moment vs. rotation curve to be symmetrical and relatively linear near $\gamma = 0$, the region where a force-feedback instrument would normally be operating. This is equivalent to requiring that the free period remain nearly constant for γ angles near 0.

The Problem

Most equations dealing with the bending of objects assume that, when the load forces are applied, the shape of the object (a beam) does not change appreciably from its un-loaded shape. For the beams supporting the floors of a building, for example, this is a reasonable assumption. A leaf spring, bent through something like 180 degrees does not meet the assumption of constant shape, and designers would typically use a Finite Element Analysis method to analyze such a spring. The problem would be solved by using successive approximation.

Lacking an FEA program I tried to see if the analysis could be done using the Microsoft spreadsheet program, Excel. The desired analysis can, indeed, be done by using a series of Excel worksheets, 'FBVsolve_Y1.14d.xls', into which all the relevant parameters are entered,. Then, the effects of changing the various dimensions can be studied as they relate to the resulting period of oscillation, linearity and stability.

The problem of determining the physical characteristics of a system of the FBV design may be broken into several parts.

1. Determine the effective boom mass value, the location of its center of mass and its radius of gyration or rotational moment of inertia. These are determined by weighing and measuring the physical boom. For the Yuma2, these values are generally known.

2. As a function of the boom rotation angle, γ, (0 = horizontal rest position) compute the coordinates of the center of mass and the spring attachment point and the angle of the spring attachment. Worksheet "Rotation" handles this. Line 6 contains the results of the computation for a particular boom angle, as defined in cell A76 of worksheet “Control” (Control!A76).

3. As a function of the leaf spring dimensions and material properties, i.e., its stiffness, compute the spring end force (magnitude and direction) and the spring end moment, given the spring-boom attachment point coordinates and angle.

This step is the most involved to describe. For computational purposes, the leaf spring is broken into 600 small but finite elements of identical length.

Beginning at the point where the spring is attached to the mounting base, the bending moment is calculated for the first element which allows the bend radius, ‘r’, to be determined in the vicinity of that point. Then knowing the location and angle of the element's starting face, the location and angle of its opposite face may be accurately computed. In turn, the ending coordinates and angles of each element are used as the starting conditions for the next. In this way, the shape of the entire spring may be built up, element by element. Worksheet "spring" does this computation, using 600 lines, one for each of the elements, and the resultant spring shape is graphed on the chart "shape".

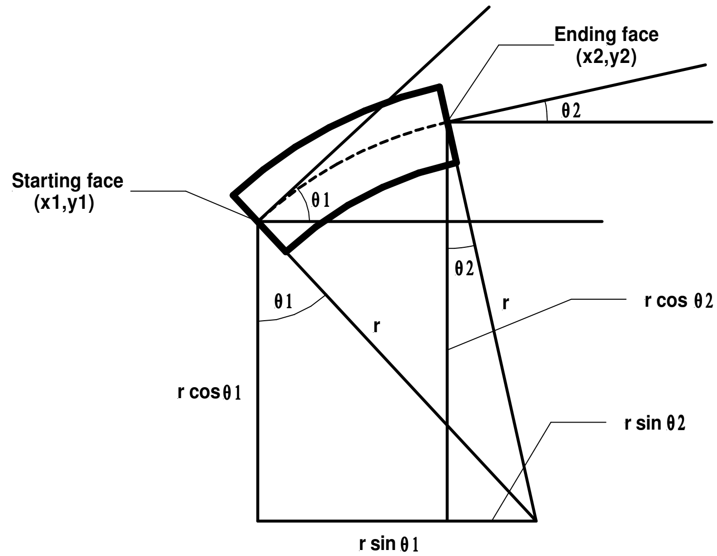
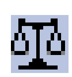


Figure 4

For convenience in calculation (avoiding circular references) it is assumed that the force is applied to the spring at the (fixed) location of the boom attachment point, which will not necessarily start out coincident with the end of the spring. This can be viewed as if the force were applied at the end of a rigid link from the attachment point on the boom and extending to the end of the spring.

However, it should be noted that the problem we have just solved is the inverse of what we need. What we wanted were the values of the force and moment given the desired spring-end position. These are obtained in the following step by using the Excel "Solver" to back-solve the problem, performing a systematic trial and error search to find the exact force and moment which will result in the required spring end location and angle. The effect of this will be to make the end of the spring coincide with its attachment point on the boom, and the imaginary link described above, disappears.

To do this for one boom angle and parameter set, click  "Run Solver". The results will appear in the "Control" worksheet, as changes to the spring end force and moment in cells C66-68, and also to any variables deriving from them. These changes will also be reflected in charts “Shape”, “Moment” and “Slope”.

4. For each of 21 boom rotation angles from +5 degrees to -5 degrees, compute the net restoring moment on the boom. This process has been automated with the macro  "Spring_Solve", which when invoked by clicking it, first adjusts the center of mass horizontal location for exact balance at $\gamma = 0$, and then goes on using the Excel Solver to compute the significant moment values for all 21 values of gamma. The results

are placed in the Results Table, to become the source data for the three charts "Net Moment", "Pivot Moments" and "Spring Moments" A copy of the important parameters is created adjacent to the Results Table, for convenience in archiving the data set.

5. Plot the net restoring moment vs. boom angle. Then compute the slope of the net restoring moment curve at the point of boom equilibrium.

Worksheet "CurFitter", generates a smoothed version of the plot of the net moment around the pivot vs. boom tilt angle, γ, which is also displayed on charts "Pivot Moments" and "Net Moment" along with the previously computed results.

6. Finally, compute the free period based on the boom's rotational moment of inertia and the slope of the net restoring moment curve.

This is calculated in worksheet "Control" and uses the slope of the net moment data obtained in step 5 and the boom's moment of inertia from step 1 to obtain the free period of the spring-mass for the selected set of parameters. The period is displayed in cell C42.

Notes on the Excel analysis system

The worksheet "Control" is where parameters are entered and where the resulting values are viewed. Cells which contain input parameters are yellow. Cells which contain key results (formulas) are blue.

When “Spring Solve” finishes calculations for all 21 values of γ, the results may be saved by clicking  "Save Plot Data". This appends the resulting data and parameter set to worksheet "Data Sets". Then the process may be repeated using different parameters. In order to help identify the data set, the date and time of the data run in cell A79 and any reference information entered by the user in Cell B79 will be included at the beginning of the saved data.

To retrieve a previously saved data set along with its parameters, go to the "Data Sets" worksheet. Highlight (click on) the blue date/time cell at the beginning of the desired data set, then click  "Load Plot Data". This will copy the selected data set and its associated parameters back to the "Control" worksheet.

Results

Of interest is how changes of only a tenth of a millimeter in the spring length can significantly affect the free period. The data below were obtained using the Yuma2 parameters and varying the spring length. We see that the free period is a function of the leaf spring length, and gets longer as the spring gets shorter until we pass the point of neutral stability (infinite period) at 3.4433 inches of length, and the period numbers jump negative. A negative period indicates that the associated configuration is unstable and its value suggests how quickly the boom will tend to move toward the end stop.

Spring Length in	Free Period Seconds	Effective torque const.- lb_f-in/deg	C.G. X Coord. - in
3.462	2.450	-0.0005528	3.042
3.460	2.593	-0.0004951	3.044
3.458	2.763	-0.0004371	3.046
3.455	3.096	-0.0003494	3.050
3.453	3.400	-0.0002906	3.052
3.451	3.814	-0.0002314	3.054
3.449	4.431	-0.0001719	3.056
3.447	5.495	-0.0001121	3.059
3.445	8.082	-0.0000519	3.061
3.444	12.502	-0.0000217	3.062
3.4433	79.288	-0.0000005	3.063
3.443	-19.940	0.0000086	3.063
3.4425	-11.976	0.0000237	3.064
3.442	-9.354	0.0000389	3.064
3.441	-7.011	0.0000694	3.066
3.440	-5.845	0.0000999	3.067
3.438	-4.607	0.0001612	3.069
3.436	-3.923	0.0002229	3.071
3.434	-3.475	0.0002849	3.074
3.432	-3.151	0.0003473	3.076
3.430	-2.904	0.0004100	3.078
3.427	-2.622	0.0005047	3.081
3.424	-2.409	0.0006002	3.085

Table 1

For each spring length selected, the center of mass location must be adjusted very slightly to obtain an exact balance when the beam is horizontal.

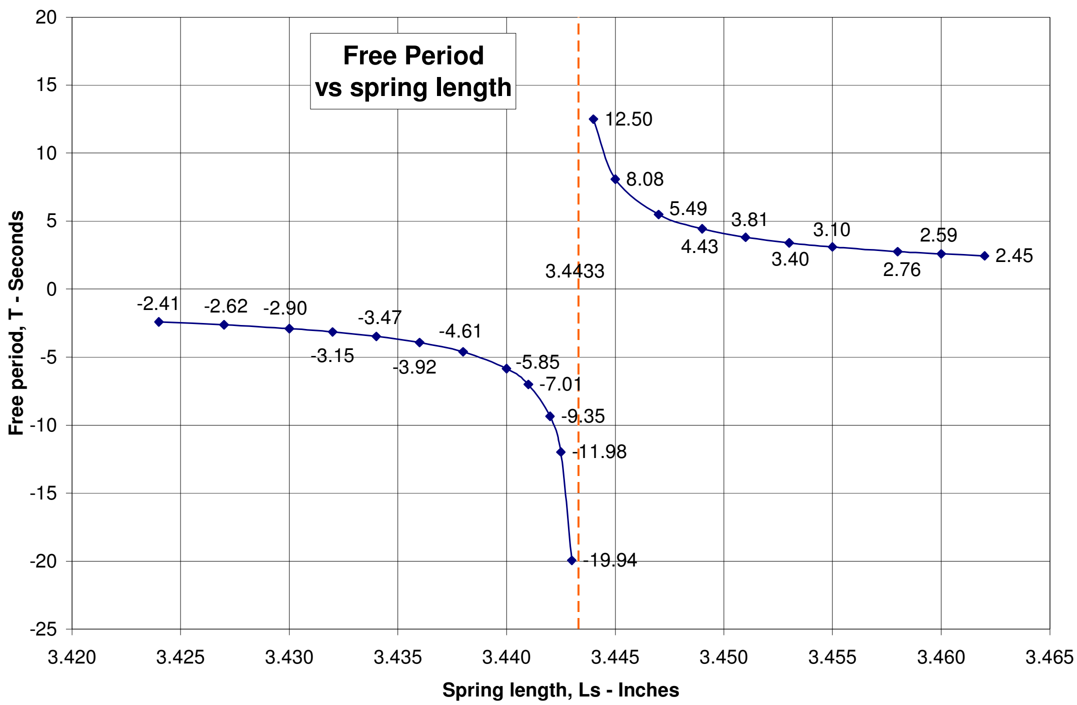


Figure 5

It should be noted that there is nothing particularly special about adjusting the spring length to vary the free period, just as there is nothing special about the spring lengths obtained. I believe that a similar looking set of data could be created by keeping the spring length constant and systematically adjusting some other dimension of the system.

In looking at the sensitivity of the system to changes in its various dimensions, it appears that many or most of the other dimensions may be just as critical as the spring length. A 0.1 mm variation from optimum makes a significant difference in the results. One is unlikely to get this astatic design working well by trial and error methods, and preliminary modeling will almost certainly have to be an essential part of the design process.

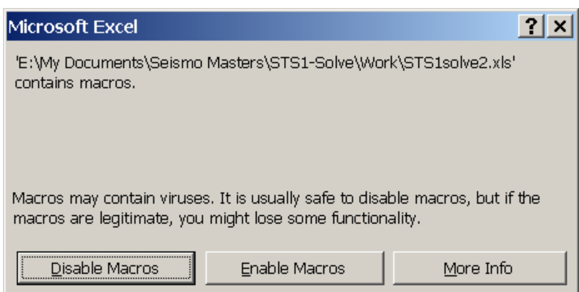
The Excel Cookbook:

Notes:

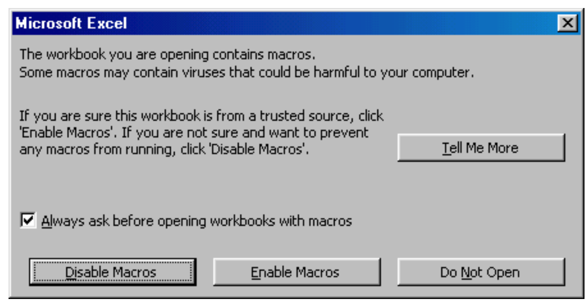
This workbook will only work with Excel 97 and later versions. It is also computationally intensive. The routine “Spring Solve” requires nearly one minute to complete, with a 2 GHz PentiumIV, which implies that “Run Solver” is requiring an average of 2.5 seconds per solution. Excel 2002, when running this workbook, was requiring 20Mb of free memory for the worksheet and the operation of its macros. With a 600 MHz Pentium III, computation was about 3 times slower. However, with a more modern computer, "Spring Solve" completes within about 10 seconds.

Getting Started:

Macro security:
Since this workbook uses VBA macros you will want to give Excel permission to run them. Some macros contain viruses, though I have tried to make sure that the ones here don’t. When opening this workbook, you should agree to enable macros unless you are just browsing.



Newer Version



Older Version

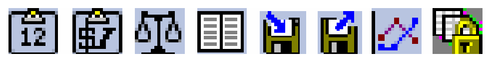
To permit enabling macros, in Excel (newer versions), with a blank worksheet, go to “Tools / Options / Security / Macro Security / Security Level” and select either Low or Medium (recommended), then OK (twice). Note that if you selected Low, Excel will never offer you the option to disable macros for *any* Workbooks you may open.

Some earlier versions of Excel, allow macros all the time. Slightly more recent versions permit you to activate a warning popup (recommended) which allows you to select whether to allow macros to run or not. In those version you can go to “Tools/ Options / General” and checkmark “Macro virus protection”. This has exactly the same effect as setting the Security Level to “Medium” as described above.

Solver:
This Workbook requires the Excel Add-in called “Solver” To determine that it will be available, Open Excel with a blank worksheet, then select Tools / Add-ins and look for “Solver Add-in” in the list and be sure that it is check marked.

However, if there is no “Solver Add-in” listed, you can expect that the first time this Workbook is opened, you will be asked to install it. That will require you to have available the original installation files for Excel, either from a CD, your hard drive or from a network. It is possible that the "Analysis Took Pak" also needs to be present and check marked.

Custom toolbar:
After opening this Worksheet, look to see that the "STS-1" custom toolbar is visible.



If not, select “View / Toolbars ” and make sure the "STS-1" entry is checked.

Worksheet locking.

All worksheets are protected, which prevents the accidental altering of important formulas. Cells which contain constants which can be altered are colored light yellow. Data may be entered in those cells, and all macros may be run without unlocking the worksheets. To unlock a worksheet, select Tools/Protection/Unprotect Sheet.

The Charts:

Shape (Updated real-time. Valid solution after running “Solver”)
This shows the shape of the spring. The X and Y coordinate scales should be kept the same to avoid distortion. It is a great place to see how the spring end forces are adjusted to get the desired end coordinates. Whenever you change a parameter, the spring end will move away from the target point (assuming the “Auto Solve” cell was set to “FALSE”). After running Solver, the end forces will have been adjusted so that the spring is connected again.

Net Moment (Plotted from data in the table generated by “Spring Solve”)
Shows the restoring moment vs. boom rotation, plotted from the difference between the gravity moment and spring moment curves, or rather, from cubic polynomial curves fitted to them. The present setup computes the moment data accurately enough for each curve to be essentially identical to its fitted approximation. A line tangent to the Net Moment curve at zero degrees rotation approximates the torsion spring constant, which is determining the free period. Also a plot of slope vs. boom rotation gives a quantitative look at the degree of nonlinearity of this virtual spring.

Pivot Moments (Plotted from data in the table generated by “Spring Solve”)
Shows the gravity and spring moments separately vs. boom rotation. These are opposing moments, but the gravity moment is plotted as its negative. Where the curves intersect is the point where the net moment is zero. At present, “Spring Solve” adjusts the center of mass location slightly, which raises or lowers the gravity moment curve so that, for the

assumed set of parameters, the intersection occurs at zero degrees boom rotation. These curves are very helpful in visualizing how geometry changes are affecting the free period.

Spring Moments (Plotted from data in the table generated by “Spring Solve”)
The spring has two effects on the boom. First it creates a force which tends to rotate the boom downward. Then its end moment, which in the default example is about four times larger, tends to rotate the boom upward. The force-induced moment, the spring end moment and their sum are plotted vs. boom rotation. It is interesting that both components of the spring moment, when varying the boom angle, change in opposite directions, so that the total spring moment change with boom rotation is rather small. Also included in the Spring Moment value is the small contribution from the flexures.

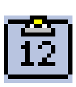
Period (Plotted from data in manually-entered table)
shows free period vs. spring length for a set of assumed parameters. It is plotted from the "Control” worksheet, "Summary, current values" data. This data for this chart were entered by hand by running “Spring Solve” for successive values of spring length and copying the results into the table.

Moment (Updated real-time. Valid solution after running “Solver”)
shows the spring bending moment vs. distance along the spring from s = 0 to L, the end of the spring. Data comes from Worksheet "Spring".

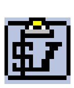
Slope (Updated real-time. Valid solution after running “Solver”)
shows the angle θ the spring makes with the world horizontal vs. distance along the spring. Where the spring attaches to the base, θ = 90 degrees, at s = 0, . If the boom is horizontal, θ = -90 degrees, at s = L, where the spring attaches to the boom. Data comes from Worksheet "Spring".

The STS-1 Command Bar:

Auxiliary buttons:



Paste Values
Pastes only copied cell value, not its formula or format.
Almost always you will want to use this instead of <ctrl>V or Edit, Paste.



Paste Format
Pastes only cell format, not its value or formula.
You will likely need this only when modifying the worksheets.

Protect All
Sets all worksheets and charts in this workbook to "Protected". If worksheets have been unprotected for editing this would normally be used before saving the workbook.

Spring solving buttons:

Run Solver
Attaches the spring end to the boom by recomputing the spring end force and moment to match the current boom position and other parameters.
You will likely need to run this after changing any parameter.

Spring Solve
Computes and records a complete set of 21 results for boom positions from -5 deg to +5 deg. using the current parameter set in Control!C2:C20, etc. The result table is saved in Control!A80:K100 and is used to create the three "Moment" graphs. A copy of the parameters used is placed to the right of the data set.

Save Plot Data
Appends the current "Plot Data" results table and parameters to the text worksheet "Data Sets"

Load Plot Data
After selecting the desired Date/Time cell (blue), retrieves the results table data from "Data Sets" into the "Plot Data" table for display in the "Moments" graphs. Sets all parameters to the values which were used to create to the data.

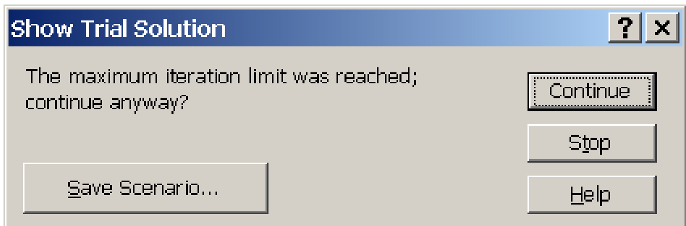
Set Chart Titles
Updates the Chart Title variables to correspond to the data sets being displayed. Use this before printing or using the charts. However it will be done automatically by "Run Solver", "Spring Solve" and "Load Plot Data"

Auto Solve
Almost any time an input parameter (yellow cell) is changed, the spring end becomes detached from its attachment point on the boom, which can best be observed in the Chart "Shape". In order to reconnect the spring one must execute the "Run Solver" macro (above), which iteratively seeks the solution. This procedure can be automated by entering "TRUE" in cell H73 of Worksheet "Control". Then, any change to a cell which causes the spring end to separate from its attachment will automatically invoke "Run Solver". By default, H73 is set "FALSE" (recommended).

Solver Convergence
Although the Excel "Solver" program usually does an excellent job of converging on the correct solution, it is possible for it to become confused if its starting conditions are badly chosen or following large changes made to the parameters. If necessary, the starting

values may be entered manually into cells C66-68 of the "Control" worksheet. These cells normally contain the values calculated in the previous "Solver" run.

To observe the sort of problem which may occur, make sure cell H73 of "Control" is "FALSE"; then try entering 0, 0, -1 for F, ϕ and Q(L). You can see on the "Shape" chart that the spring is now curving off the chart to the left. Then click the "Run Solver" button. After 100 iterations, Solver will pause, and you should click "Stop" in the pop up window which appears.



Now look at "Shape" again.

This sort of problem can be avoided by ensuring that the starting conditions have the spring bending in the correct direction and ending in the general vicinity of the boom attachment point. Normally that will be the case if changes made to the parameters are not too large. However if there is a problem, in C66:C68 you can try entering 0 for F and φ and then find a value for Q_N which has the spring bending to place its end in the general neighborhood of Xa,Ya. Note that entering 0, 0, 0, creates undefined values which are associated with having specified an infinite bending radius.

Known bugs:

1) If you start to close the worksheet, but click "cancel" on the "Do you want to save changes" pop-up, the special "STS-1" toolbar disappears.

Workaround to recover it: Select Tools / Customize / Toolbars Then scroll down and restore the checkmark for "STS-1".

Some Random Thoughts:

Thin beams: A wide, thin, spring behaves slightly differently when bent compared with a piece of steel that has, for example, a square cross section. The usual beam-bending equations would be accurate for the square beam, but not for the leaf spring. The latter will appear to be stiffer than predicted by a factor of about 9%, or to be more precise by a factor of $1/(1-\nu^2)$ where ν is Poisson's ratio, which for steel is about 0.29. The easiest way to incorporate this correction into the bending equations is to assume an increased value for the modulus of elasticity of the spring material by multiplying by that factor. If the stated modulus of elasticity for the spring material is E, then the corrected value, called E_1 in the spreadsheet, equals $E/(1-\nu^2)$ or for steel, E/ 0.916 .

Temperature coefficient:
For instruments like the Yuma, which have frequency responses with two zeros at zero and a double pole at their low-frequency corner, will, when experiencing slow temperature changes, exhibit an apparent velocity offset which is proportional to the rate of temperature change seen by the spring. This will be in the amount of 6.9E-5 K_{ET} T_L^2 μm/s per ºC/hour, where K_{ET} is the temperature coefficient of the spring's modulus of elasticity in parts per million per ºC and T_L is the period of the instrument's low frequency response corner in seconds. For the Yuma $K_{ET} \approx -240$ ppm/ºC and $T_L = 50$ seconds, which predicts an output offset of -41.4 μm/s per ºC/hour.
However, its response to a temperature change will be essentially zero, once the spring temperature has stabilized.

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Bibliography:

1 Wielandt, E. & Streckeisen, G., 1982.
The leaf-spring seismometer: design and performance, Bull. Seism. Soc. Am., 72(6), 2349-2367.

2 Morrissey, S.-T., 26 Mar 2000
Correspondence archived on Public Seismic Network
http://psn.quake.net/info/stm-mail.zip
http://www.eas.slu.edu/People/STMorrissey/index.html

Appendix I

The Solver Model

The macros “Run_Solver” and “Spring_Solve” both use the “Solver” add-in program. The control parameters passed to “Solver” are located on the “Control” worksheet in the named range “Solver_Model” located in cells H66-71. The data in those cells is as follows:

Cell	Contents	Meaning
H67	=MIN($F$72)	Minimize F72 (θ(L) error squared)
H68	=COUNT($C$66:$C$68)	By varying cells C66-68
H69	=$C$70=Control!$D$70	Constraint: X(L) = its target value
H70	=$C$71=Control!$D$71	Constraint: Y(L) = its target value
H71	={60,500,0.0000000001,0.00000000001,FALSE,FALSE,TRUE,2,2,1, 1E-20,FALSE}	

Cell H71 contains multiple parameters:

60	Max time – sec.
500	Max number of Iterations
0.0000000001	Precision
0.0000000001	Tolerance
FALSE	Assume linear (not)
FALSE	Assume non-negative (not)
TRUE	Use automatic scaling
2	Estimates: Quadratic (selection 2)
2	Derivatives: Central (selection 2)
1	Search: Newton (selection 1)
1E-20	Convergence
FALSE	Show iteration results (not)

These values have been found to generally work pretty well. On a very slow computer, it may be necessary to reduce “Max Time” and “Iterations” to permit the process to abort if it is taking too long.

Appendix II

To Understand the Math:

It is not necessary to understand the math, physics or mechanics which were used to create the spreadsheet in order to use it, but the more background the reader has in certain areas, the better the problem can be understood.

To begin with, the over all design is analyzed as a torsion pendulum, so it is useful to have seen something of the physics of such a device. In particular, the analysis considers this pendulum to be a distributed mass, rather than a point mass, which involves such concepts as the Radius of Gyration. A beginning physics book would cover these.

Secondly, a basic understanding of forces, force vectors and moments is fundamental to the analysis. It is useful to understand how to interpret a free-body diagram, such as in fig. 3. This involves understanding the relationship between moments and forces and understanding that, when at rest, the forces and moments on such a body must both sum to zero. These would be covered in a beginning book on Mechanics or Strength of Materials.

Also, such books would cover the simple beam-bending equations used to compute the bending radius of each element of the spring, as shown in figure 6, which uses concepts such as material Elastic Modulus, and properties of the spring cross-section shape, such as the section ‘Area Moment of Inertia’ (which incidentally has little to do with physical inertia).

And algebra and trigonometry are used throughout.

Appendix 1, Macros

```
Public SolverFlag As Boolean

Private Sub Auto_Open()

   Toolbars("STS-1").Visible = True
   Toolbars("STS-1").ToolbarButtons(3).OnAction = "RunSolver"
   Toolbars("STS-1").ToolbarButtons(4).OnAction = "SpringSolve"
   Toolbars("STS-1").ToolbarButtons(5).OnAction = "SavePlotData"
   Toolbars("STS-1").ToolbarButtons(6).OnAction = "LoadPlotData"
   Toolbars("STS-1").ToolbarButtons(7).OnAction = "SetChartTitles"
   Toolbars("STS-1").ToolbarButtons(8).OnAction = "ProtectAll"
   ProtectAll
   CheckSolver

End Sub
'-------------------------------------------------------

Private Sub Auto_Close()

   Toolbars("STS-1").Visible = False

End Sub
'-------------------------------------------------------

Private Sub CheckSolver()

  If Application.AddIns("Solver Add-in").Installed = False Then _
     AddIns.Add("solver.xla").Installed = True

End Sub
'-------------------------------------------------------

Private Sub ProtectAll()
   Dim Sheet As Worksheet
   Dim ChartPage As Chart

Application.ScreenUpdating = False

'Protect all Worksheets But allow modification by programs
   For Each Sheet In Worksheets
      Sheet.Protect UserInterfaceOnly:=True
   Next Sheet

'Protect all Charts
```

```
    For Each ChartPage In Charts
        ChartPage.Protect
    Next ChartPage
Application.ScreenUpdating = True
End Sub
 '-------------------------------------------------------

Private Sub SpringSolve()
'
' Keyboard Shortcut: Ctrl+l
'
On Error GoTo ErrorHere
SolverFlag = True

Worksheets("Control").Activate

Range("Result_Table").ClearContents
Range("Calculated").ClearContents
Range("Param_Table").ClearContents
Range("Result_Table2").ClearContents

' Put a copy of the Paramater Values in the table
Range("Params").Copy
Range("Param_Table").Range("A1").PasteSpecial (xlPasteValues)

'Boom at Zero
'
'   Compute spring for Gamma = 0
Range("Results").Cells(1, 1) = 0
'
SolverLoad loadArea:=Range("Solver_Model")
SolverSolve UserFinish:=True

'   Adjust Center of Mass 'Xc' for zero Net Moment

For I = 1 To 10
CGX = Range("Xc") * (0.995 + 0.01 * Rnd)
Range("Xc") = CGC
Range("dQp").GoalSeek Goal:=0, ChangingCell:=Range("Xc")
If Range("dQp") = 0 Then Exit For
Next

'Calculate Moments, etc. for all 21 values of Gamma

For Gamma = 0 To 5 Step 0.5
    Range("Results").Cells(1, 1) = Gamma
```

```
    SolverSolve UserFinish:=True
    Res_Row = 11 - (2 * Gamma)
    Range("Results").Copy
    Range("Result_Table").Range(Cells(Res_Row, 1), Cells(Res_Row, 11)). _
        PasteSpecial (xlPasteValues)
    Range("Results").Cells(1, 1).Select
Next Gamma

Range("Result_Table").Range("B11").Copy
Range("EndForces").Range("A1").PasteSpecial (xlPasteValues)
Range("Result_Table").Range("C11").Copy
Range("EndForces").Range("A2").PasteSpecial (xlPasteValues)
Range("Result_Table").Range("F11").Copy
Range("EndForces").Range("A3").PasteSpecial (xlPasteValues)

For Gamma = -0.5 To -5 Step -0.5
    Range("Results").Cells(1, 1) = Gamma
    SolverSolve UserFinish:=True
    Res_Row = 11 - (2 * Gamma)
    Range("Results").Copy
    Range("Result_Table").Range(Cells(Res_Row, 1), Cells(Res_Row, 11)). _
        PasteSpecial (xlPasteValues)
    Range("Results").Cells(1, 1).Select
Next Gamma

    Range("Results").Cells(1, 1) = "0"
Range("Result_Table").Range("B11").Copy
Range("EndForces").Range("A1").PasteSpecial (xlPasteValues)
Range("Result_Table").Range("C11").Copy
Range("EndForces").Range("A2").PasteSpecial (xlPasteValues)
Range("Result_Table").Range("F11").Copy
Range("EndForces").Range("A3").PasteSpecial (xlPasteValues)

'Range("Qcoeff").Range("A1:A4").Copy
'Range("Result_Table2").Range("A1:A4").PasteSpecial (xlPasteValues)
'Range("Qcoeff").Range("B1:B4").Copy
'Range("Result_Table2").Range("A5:A8").PasteSpecial (xlPasteValues)
Range("T0").Copy
Range("Result_Table").Range("L19").PasteSpecial (xlPasteValues)
Range("Ks").Copy
Range("Result_Table").Range("L20").PasteSpecial (xlPasteValues)
Range("gFp").Copy
Range("Result_Table").Range("L21").PasteSpecial (xlPasteValues)

'Put the value of the adjusted mass CG in table
```

```
Range("Params").Range("A5").Copy
Range("Param_Table").Range("A5").PasteSpecial (xlPasteValues)

'Range("Param_Table").Cells(21, 1).PasteSpecial (xlPasteValues)

'Set Chart Titles to reflect current parameters.
SetChartTitles
Range("Calculated") = Now
SolverFlag = False

'MsgBox "Spring solution complete."

Exit Sub

ErrorHere:
MsgBox Error(Err.Number)
''Resume
End Sub
'-------------------------------------------------------

Sub RunSolver()
'
' RunSolver Macro
'
On Error GoTo ErrorHere

' For faster operation, don't update screen while running.
Application.ScreenUpdating = False

' Flag so that AutoSolve doesn't execute this recursively as cells are updated.
SolverFlag = True

Worksheets("Control").Activate

SolverLoad loadArea:=Range("Solver_Model")
Range("Restart").Copy
Range("EndForces").PasteSpecial (xlPasteValues)
SolverSolve UserFinish:=True

'Set Chart Titles to reflect current parameters.
SetChartTitles

Application.ScreenUpdating = True
SolverFlag = False
Exit Sub
```

```
ErrorHere:
MsgBox Error(Err.Number)
Resume
End Sub
'------------------------------------------------------

Private Sub LoadPlotData()
'
' For faster operation, don't update screen while running.
Application.ScreenUpdating = False
SolverFlag = True

' Check that "Data Sets" is the active sheet
If ActiveSheet.Name <> ("Data Sets") Then Exit Sub

' Check that active cell is colored
If Selection.Interior.ColorIndex <> 28 Then Exit Sub

'Selection.Copy
'Range("Calculated").PasteSpecial Paste:=xlValues

'Select copy range as relative
   ActiveCell.Offset(0, 0).Range("A1:B1").Select
   Selection.Copy

   Sheets("Control").Range("Calculated").Range("A1") _
      .PasteSpecial Paste:=xlValues

   ActiveCell.Offset(1, 0).Range("A1:L21").Select
   Selection.Copy

   Sheets("Control").Range("Result_Table").Range("A1") _
      .PasteSpecial Paste:=xlValues

'       ActiveCell.Offset(0, 0).Range("M1:M10").Select
'    Selection.Copy
'
'    Sheets("Control").Range("Result_Table2").Range("A1") _
      .PasteSpecial Paste:=xlValues

'Copy saved Param Values to "Params"
   Sheets("Control").Range("Param_Table").Range("A1:A18").Copy
   Range("Params").PasteSpecial Paste:=xlValues

'Set spring end conditions
   Range("Result_Table").Range("B11").Copy
```

```
    Range("EndForces").Range("A1").PasteSpecial Paste:=xlValues

    Range("Result_Table").Range("C11").Copy
    Range("EndForces").Range("A2").PasteSpecial Paste:=xlValues

    Range("Result_Table").Range("F11").Copy
    Range("EndForces").Range("A3").PasteSpecial Paste:=xlValues

'Set current gamma to zero
    Range("Results").Range("A1").FormulaR1C1 = "0"

'Set Chart Title variables
    SetChartTitles

Application.ScreenUpdating = True
SolverFlag = False

End Sub
'-------------------------------------------------------

Private Sub SavePlotData()

Application.ScreenUpdating = False
Worksheets("Control").Activate

CalcDate = Range("Calculated")
Comment = Range("Comment")
Range("Result_Table", "Param_Table").Copy
Worksheets("Data Sets").Activate

LastRow = Cells.Find(What:="*", _
    SearchDirection:=xlPrevious, _
    SearchOrder:=xlByRows).Row
Worksheets("Data Sets").Cells(LastRow + 2, 1).PasteSpecial xlPasteValues
'Range("Result_Table2").Copy
'Worksheets("Data Sets").Cells(LastRow + 2, 13).PasteSpecial xlPasteValues
Cells(LastRow + 1, 1) = CalcDate
Cells(LastRow + 1, 1).Interior.ColorIndex = 28
Cells(LastRow + 1, 2) = Comment
Worksheets("Control").Activate
Application.ScreenUpdating = True

End Sub
'-------------------------------------------------------

Private Sub SetChartTitles()
```

```
Application.ScreenUpdating = False
Worksheets("Control").Activate


SpringLenData = Range("Param_Table").Cells(3)
SpringLen = Range("Params").Cells(3)
Gamma = Range("A76")

With Charts("Net Moment")
   .Unprotect
   .ChartTitle.Text = "Net Pivot Moment vs Boom Angle" _
      & vbCr & "L = " & Format(SpringLenData, "0.0000") & " in"
   .Protect DrawingObjects:=True, Contents:=True, Scenarios:=True
End With

With Charts("Pivot Moments")
   .Unprotect
   .ChartTitle.Text = "Pivot Moments vs Boom Angle" _
      & vbCr & "L = " & Format(SpringLenData, "0.0000") & " in"
   .Protect DrawingObjects:=True, Contents:=True, Scenarios:=True
End With

With Charts("Spring Moments")
   .Unprotect
   .ChartTitle.Text = "Spring Moments vs Boom Angle" _
      & vbCr & "L = " & Format(SpringLenData, "0.0000") & " in"
   .Protect DrawingObjects:=True, Contents:=True, Scenarios:=True
End With

Charts("Shape").Unprotect
With Charts("Shape").ChartTitle
   .Text = "Spring Shape" & vbCr & "L = " _
      & Format(SpringLenData, "0.0000") & " in" _
      & "     g = " & Format(Gamma, "0.0") & " Deg."
   .Characters(Start:=32, Length:=1).Font.Name = "Symbol"
   .Characters(Start:=14, Length:=99).Font.Size = 14
End With
Charts("Shape").Protect DrawingObjects:=True, Contents:=True, Scenarios:=True

Charts("Moment").Unprotect
With Charts("Moment").ChartTitle
   .Text = "Spring Interior Moments Q(s)" _
      & vbCr & "L = " & Format(SpringLenData, "0.0000") & " in     " _
      & "g = " & Format(Gamma, "0.0") & " Deg."
   .Characters(Start:=48, Length:=1).Font.Name = "Symbol"
```

```
    .Characters(Start:=30, Length:=32).Font.Size = 14
End With
Charts("Moment").Protect DrawingObjects:=True, Contents:=True, Scenarios:=True

Charts("Slope").Unprotect
With Charts("Slope").ChartTitle
    .Text = "Spring Tangent Angle q(s)" _
        & vbCr & "L = " & Format(SpringLen, "0.0000") & " in     " _
        & "g = " & Format(Gamma, "0.0") & " Deg."
    .Characters(Start:=22, Length:=1).Font.Name = "Symbol"
    .Characters(Start:=45, Length:=1).Font.Name = "Symbol"
    .Characters(Start:=26, Length:=32).Font.Size = 14

End With
Charts("Slope").Protect DrawingObjects:=True, Contents:=True, Scenarios:=True

Application.ScreenUpdating = True

End Sub
```