Using the Excel Poles&Zeros workbook

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Note: When opening the workbook 'P&Z.xls' if you are using the default security settings, you will be asked whether to Enable Macros. You should enable them. The workbook contains one macro which changes <ctrl>v so that it pastes only cell values, but not their formats.

Poles and Zeros represent points on a plane (the S-plane) and so have two dimensions. The X, horizontal coordinate is called the real part and the Y, vertical coordinate is called the imaginary part. In general, the frequency response of any instrument may be described by a mathematical (polynomial) expression, with its numerator being a product derived from all the instrument's zeros, divided by an expression which is a product derived from all its poles.

For our purposes all poles and zeros will have a negative real part. They appear in two forms, the first being single (Real) poles and zeros, which always have an imaginary coordinate of zero, and which correspond to electrical R-C and R-L circuits. Or they appear as second-order pairs which describe resonant systems, such as R-L-C circuits and damped spring-mass systems. For under-damped systems, which will have a damping factor, $\zeta < 1$, the poles and zeros occur in complex conjugate pairs, both having the same negative value for the real part, with their imaginary coordinates being ± some value relating to frequency and damping, or for over-damped systems, as real pairs, both with imaginary parts = 0.

Log amplitude vs log frequency plots (Bode plots[1]), such as the one on page 3, may be broken up into sections having approximately constant slope which appear as relatively straight lines. A slope of +1 represents a region in which the amplitude is increasing by a factor of 10, 20dB or 1 decade, for each decade of increasing frequency. A slope of +2 increases by a factor of 100, 40dB or 2 decades, per frequency decade, while a slope of 0 is constant with frequency.

In frequency-response plots, each real pole or zero defines a frequency where the slope of the plot changes, with a pole causing the slope to decrease by 1 unit and a zero causing it to increase by 1. Second-order pairs of poles cause the slope to decrease by 2 units, and depending on the associated damping factor, can create a curve which has a resonant peak. A second-order pair of zeros causes the slope to increase by two units and can exhibit a notch in the response. Zeros at zero frequency define the initial slope of the graph at the lowest frequencies, with one zero at zero giving an initial slope of +1, two zeros giving +2, etc.

The frequency response of an instrument is completely defined by a list of all its poles and zeros, along with a single additional number, which is required to pin down its mid-band sensitivity.

The worksheet 'P&Z' provides four tools for working with poles and zeros

(1) allows a list of up to 9 poles and 5 zeros to be entered in the yellow cells and from that it computes the total amplitude at various frequencies for that set, providing a graph on chart 'Amplitude' of the frequency response which results from the given list. Of mostly academic interest, it also provides an overview of the S-plane, on chart 'S-Plane', plotting their two-dimensional locations.

(2) tabulates the inverse of the net magnitude value of the entire pole-zero constellation at a selected frequency, F_0, which provide the normalization multiplier required to obtain a unity amplitude response at that frequency.

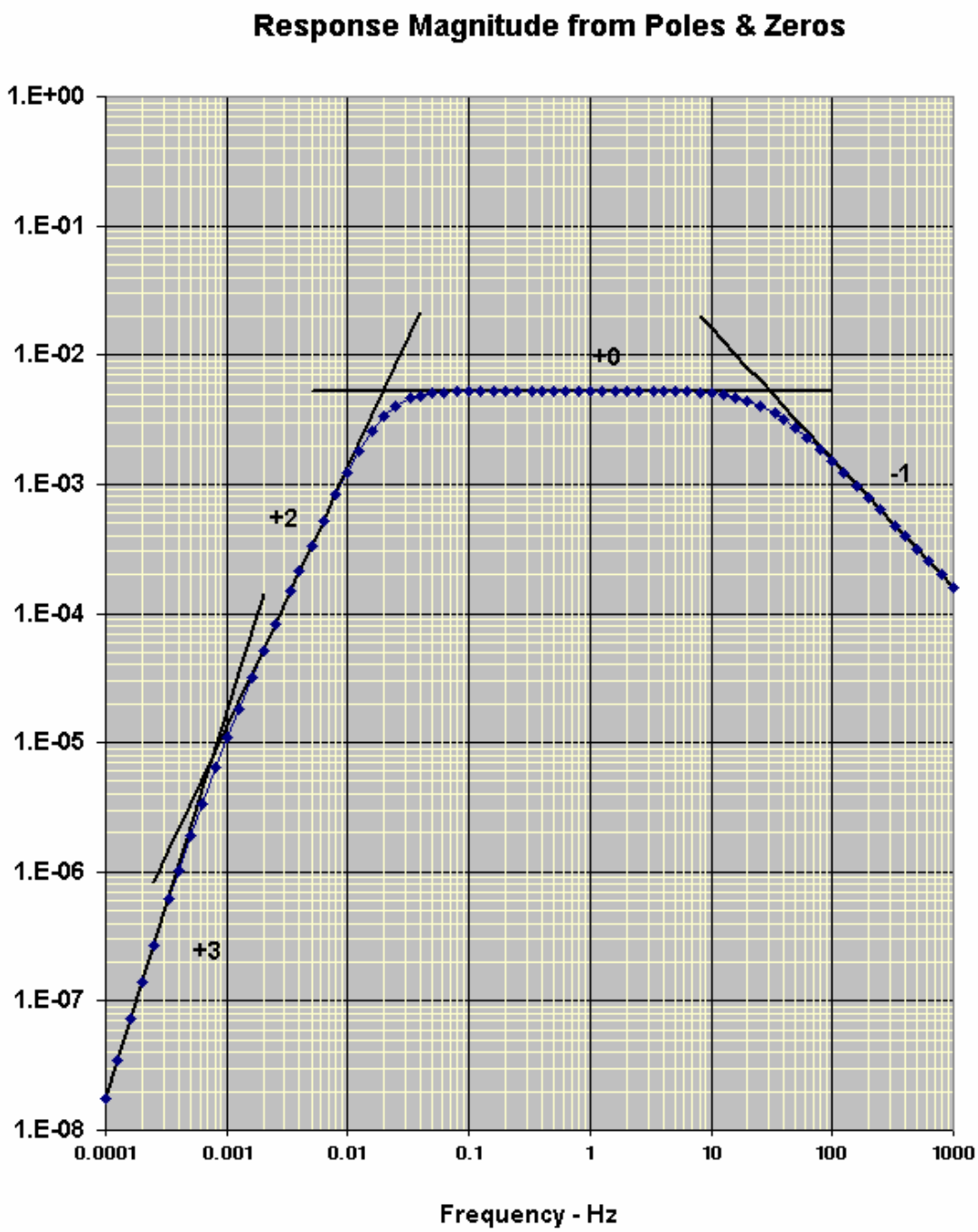
(3) computes the counts/Volt and Volts/count for various digitizers, given their maximum input voltages and numbers of bits.

(4) allows pole and zero components to be computed, by entering their "corner" frequency or its period, in one of the yellow cells, or by entering one of those plus a damping factor for second-order pole or zero pairs. It also solves the inverse problem, computing frequency and period in terms of the pole or zero component values, additionally providing the damping factor value for second-order pairs. Note that all valid second-order pairs must either have both their imaginary components = 0 (over damped) or their real parts must be exactly equal and their imaginary parts exactly equal but of opposite signs. (under damped).

Finally, the worksheet 'Instruments' is a place for archiving the poles, zeros and gain factors for specific instruments, where is also computed the 'CONSTANT' normalization factor associated with each instrument, as required by a noise-modeling program.

The worksheets and charts are protected, so that only the yellow cells may be altered. This prevents accidentally changing the formulas contained in the other cells. If protected data must be altered, you can select 'Tools' > 'Protection' > 'Unprotect Sheet' which will allow all cells to be edited. It may be a good idea to keep a copy of the original workbook, just in case something important gets altered by accident.

Note that cells and ranges of cells may be copied and pasted into yellow cells rather than having to type them. When pasting, to avoid altering the format of the recipient cell(s), right click on that cell, then either select 'Paste Special' > 'Values' > OK. or just use <ctrl>v to paste. That will assure that only the copied value is pasted, without its format.



This is a frequency plot of the net magnitude associated with the poles and zeros which describe the response of a particular broadband vertical seismometer, here, Napa#1 plus a preamp.

In this example there are three zeros at 0 frequency which gives the response curve an initial slope of +3 decades per frequency decade. A single pole in the preamp at 0.00077 Hz (0.77mHz) reduces the slope to +2. A conjugate pair of poles at about 50.1 seconds, which define a damping factor of 0.8, reduce the slope to zero, creating the flat portion of the instrument's velocity response. Another single pole at 30Hz starts its response falling with -1 slope.

It may be useful to change the data so that their mid-band amplitude value is 1.0. To do that, in section (2) of worksheet 'P&Z' we can compute k_v for some frequency, F_0, in the middle of the flat part of the response. We see that for F_0 = 1.0 Hz, k_v = 188.6..... and by multiplying all the data points by that number the flat part of the resulting curve will have a magnitude close to 1.0 .

An exercise in using Poles and Zeros.

Let's look at how we might build up the velocity response of a simple broadband instrument.

Assume we know that its response has a low-frequency corner at 20 seconds, falling with a slope of 2 below that, and that it has a high frequency corner at 15Hz, above which it falls with a slope of -1.

You will be going back and forth between the P&Z tab, where you will calculate and enter poles and zeros, and the Amplitude tab where you can look at the computed magnitude curve.

Let's clear the deck. In section (1) in the yellow cells under 'Zeros - Rad./Sec' and 'Poles - Rad./Sec." clear all the values. It will help you if you know how to select blocks of cells in Excel. To clear an area quickly, first select that block and then use the 'Delete' key.

Now go to chart 'Amplitude'. With no poles or zeros the response function magnitude is simply constant with a value of 1.

To put in a single zero at zero frequency, enter '0' in the both the Real and Imaginary columns of the first row of yellow cells in the 'Zeros' section. Now we see that the response amplitude is a line rising at a rate of +1 decade per frequency decade. Note that it has the value 1 at a frequency of $1/2\pi$ (=0.15915....Hz), which has a period of 2π seconds. In general, the spreadsheet does its computations using angular frequency ω, radians per second, where frequency in radians per second = 2π * frequency in Hz. So our response has a value of 1 at $1/2\pi$ Hz which is a frequency of 1 radian per second.

Add a second zero at zero. Type '0' in the two yellow cells in the second row of the 'zeros' section. Now the amplitude rises at +2 decades per frequency decade, again equal to 1 at $1/2\pi$ Hz.

To make our low frequency corner requires adding a second-order conjugate pair of poles to flatten the curve at 20 seconds. This represents a spring-mass-like response and we will set its damping factor at 0.7, close to the "ideal" value of $1/\sqrt{2}$. But first we need to compute the real and imaginary parts for the poles we need. In section (4) 'Pole-Zero Calculators, under 'Second-Order Pole-Zero Pairs. In the yellow cell for T_0-Sec. enter 20, and in the yellow cell to its right, enter **.**7 for the damping. Note that if we knew the corner frequency rather than its period, we could use the calculator two rows down. In the four cells in two rows of the 'Real' and 'Imaginary' columns you should find the real values of -0.2199... and imaginary values of ± 0.2243....

Now to enter those two poles into section (1), Select the four cells containing the values we just calculated and press <ctrl>c to copy them. Or you can use 'Edit' > 'Copy' or right-click and 'Copy'. Then click on the first yellow cell in the 'Real' column of the 'Poles - Rad./Sec.' section of (1) and press <ctrl>v to paste their values into four yellow cells. Note that in this workbook <ctrl>v has been modified so that it pastes only the values of the cells which were copied, not their formulas, cell colors or borders. If you prefer you could paste values by using 'Edit' > 'Paste Special' > 'Values' > OK or you could right-click on the target cell, then select 'Paste Special' > 'Values' > OK.

If you got the 4 values for the pole-pair entered successfully, you should see that the curve on the amplitude chart has been flattened at 0.05Hz (20 seconds) to a value of 1.

Now for the high-end corner, we will use the Pole-Zero Calculator to compute a pole for a single-pole high-frequency corner at 15 Hz. In the section for 'Single (Real) Pole or Zero, in the yellow cell in the 'f_0 - Hz' column, enter 15. Note that if we knew period, rather than frequency, we could use the calculator in the row below. Now select and copy the two cells it computed in the 'Real' and 'Imaginary' columns (the imaginary cell should be 0). Go to section (1) and select the first empty yellow cell in 'Real' column of the 'Poles' section and use <ctrl>v to paste in the real and imaginary parts of the computed pole.

The amplitude response should now fall with a -1 slope with a corner frequency of 15 Hz and a mid-band amplitude around 0.0106.

If you wanted to go on to use the data from these poles and zeros to describe the response of a real instrument you would have to scale its amplitude properly. First you should scale the collection of poles and zeros to have an amplitude of 1 in the flat portion of the curve. To do that, go section (2) on the P&Z worksheet, and enter in the 'F_0 - Hz' column a frequency value which is in the middle of the flat portion of the amplitude curve. In this example 1Hz looks good. If the response curve obtained from the poles and zeros is multiplied by A_0 , in this example x 94.452..., the resulting P&Z amplitude curve will have a value of approximately 1 throughout its flat part.

To get the entire instrument response, you would need to multiply k_v by the instrument's mid-band sensitivity or generator constant, S_v. Assume that it is 1500 Volts per meter/sec. And if you wanted the curve to be in counts per meter/second, you could use section (3) for the kind of digitizer you were using. Enter its maximum input voltage and number of bits to compute its counts/Volt and then multiply the previous result in Volts per m/s by that, giving you the curve in counts per m/s. The three scaling numbers we found, k_v , S_v and the digitizer counts/V could all be multiplied together to calculate a single normalizing factor to get from poles and zeros to the instrument response in counts/m/s. In this example, if you were using a 10V, 16-bit A/D, that factor would be 94.452 * 1500 * 3276.8 = 4.642E+8 counts per m/s

Now suppose you wanted to see what the instrument response looks like in terms of Acceleration, m/s^2 instead of Velocity, m/s . They say that acceleration is the time derivative of velocity, so to get acceleration you need to differentiate something, somehow, with respect to time. Using poles and zeros to describe the response makes that trivial, no calculus required. To see the response with respect to an acceleration input, simply delete one of the zeros at zero, one of the 0,0 pairs. To get the response with respect to displacement, you can see the result of integrating, by just adding a 0,0 pair. Sometimes calculus isn't so hard.

The 'Amplitude' chart attempts to set its axis ranges to match the data, but they don't always adjust themselves well enough. If you needed to fix that, select 'Tools' > 'Protection' > 'Unprotect Sheet' , then right-click each axis in turn and select 'Format Axis' > 'Scale' tab and change the Minimum and Maximum values of that axis and the value where the other axis crosses to something better.

This kind of data is used in dataless SEED file headers. IRIS has a program, PDCC which helps create those[2]. In its user manual you can see how really complicated it can get when you try to make a format which will accurately describe all conceivable combinations of instruments, amplifiers, digital filters and digitizers. However, the poles and zeros, normalizations and instrument sensitivities are essentially what you have already been using here, just with many more options.

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[1] http://lpsa.swarthmore.edu/Bode/Bode.html

[2] http://www.iris.edu/pub/programs/pdcc/PDCC_3.8_User_Manual.pdf