Analysis of the STS-1 Vertical Sensor

The Mechanical System

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Introduction

The Streckeisen STS-1 vertical force-feedback seismic sensor uses an interesting and unusual mechanical configuration. On the surface, it does not look as if it should have particularly impressive performance. Its effective boom length is quite short, a little over 2 inches, and its mass is a modest 0.6 Kg. However the STS-1 would appear to have been a venerable and well respected instrument, which commanded a $five-figure price. I was interested in how such excellent performance could be obtained with this mechanical design and if possible, derive a general method for analyzing a fixed-end astatic spring.

The STS-1 Geometry

The geometry does appear to have some attractive features. The boom is made of a pair of parallel metal plates, bridged by a few spacers. With the leaf spring contained between the boom side plates, a very compact design is achieved, as opposed to using a solid boom, which must have its spring located entirely underneath, which then requires the boom to be positioned well above the mounting base. It also appears in the STS-1, that a smaller, lighter spring and boom structure may be used while still maintaining good rigidity, thus moving the mechanical resonances up to higher frequencies. The frequency

Fig. 1
Rear view

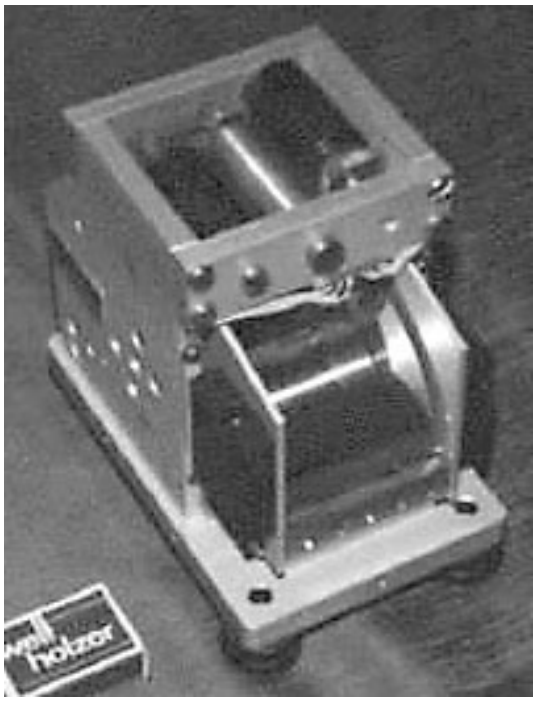


Fig. 2
Front view showing boom and seismic mass blocks

of these resonances will be a significant limiting factor on the design of a force-feedback seismometer. Also, the boom design permits the forcing and sensing elements to be located up away from the seismic mass and spring. In the STS-1 they are mounted well above the boom assembly, where any stray heat generation should be the least harmful.

The STS-1 astatic spring configuration has the leaf spring rigidly attached at one end to the base plate and at the other, to the boom. This configuration when compared with a flexibly attached spring, such as is used in S-T Morrissey's STM-8_2, has features which are advantages, but which at the same time are its disadvantages. Instead of the single degree of freedom, which can be characterized by force magnitude vs. length, the fixed-end spring has three degrees of freedom; for example the force magnitude -F, force direction ϕ, and spring end-moment -Q(L); which may be related to the spring-end x coordinate X(L), y coordinate Y(L), and end direction θ(L). The fixed-end spring thus has three parameters which may be varied by changing the system geometry. It could be hoped that this would permit better optimization of the system characteristics. On the other hand, there are many more parameter variations to be examined, and arriving at the "best" or even an acceptable combination by experimentation alone is likely to be extremely difficult.

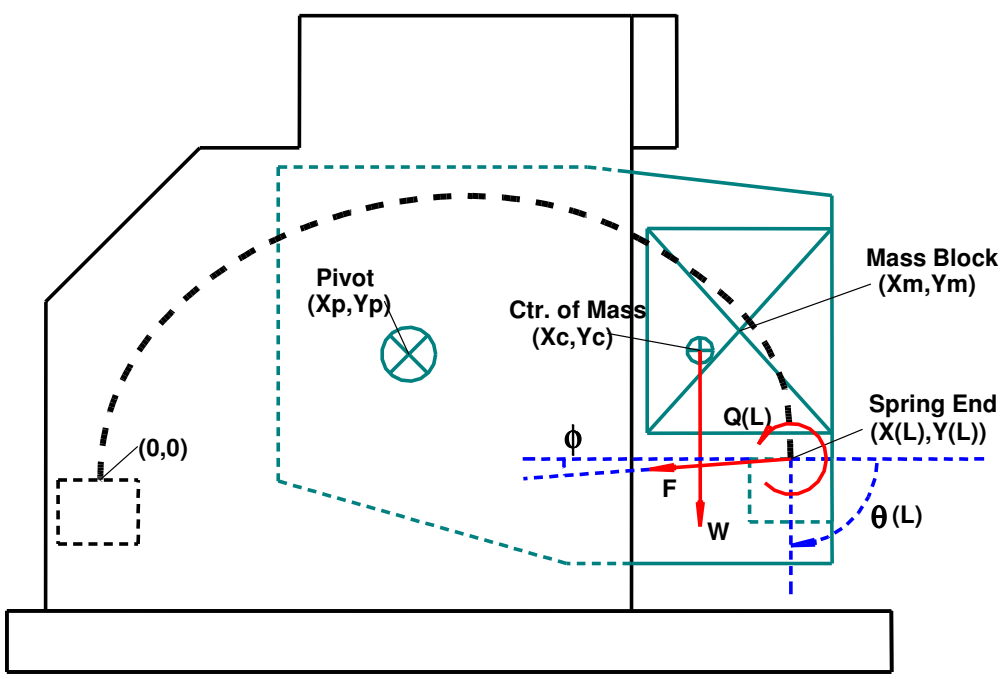


Fig. 3

Analysis

To determine the natural period, it is easiest to analyze the boom system as a rotating spring-mass, much like the balance wheel of a mechanical alarm clock. One element of the system is a distributed mass rotating about an horizontal axis, consisting of the seismic mass blocks and all the boom elements, and described by its rotational moment of inertia. Along with the mass is the equivalent of a torsion spring. This is a "virtual"

spring, arising from the sum of the moments acting on the boom, whose magnitude is approximately proportional to angular deflection, and which acts in a direction tending to restore the boom to its horizontal rest position.

The virtual spring is the resultant of two competing moments which both vary slightly with boom rotation, one from the gravity force W acting on the center of mass, and tending to rotate the boom downward (CW), the other from the leaf spring, acting upward (CCW). This virtual spring may be approximated by a rotational spring constant, defined as the (assumed constant) rate at which the net restoring moment changes per unit of boom rotation. Once we have determined the moment of inertia and effective spring constant, the free period of oscillation is easily computed.

In order to observe the nature of the restoring moment, we can compute the sum of the spring and gravity moments, and plot it vs. the up-down boom rotation angle γ. For a workable design, we want two conditions to be fulfilled. First, for equilibrium, the net restoring moment should be zero when the boom is in its horizontal rest position at $\gamma = 0$.

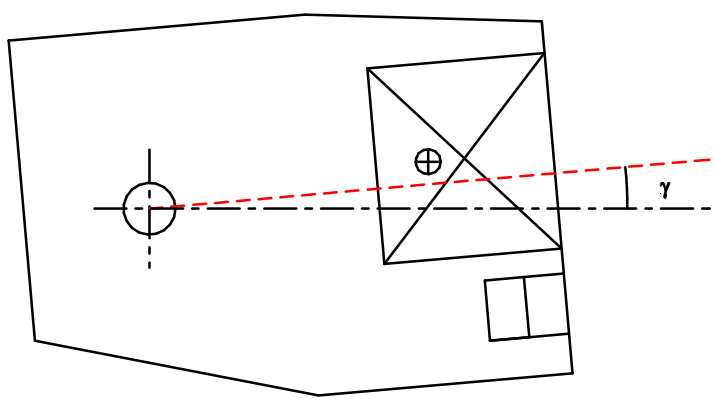


Fig. 4

Second, in order to have a stable equilibrium, the upward moment should increase as the boom rotates downward, and then reverse and become a net downward moment as the boom is rotated above zero degrees. This is of significance because it is easy to find configurations where the latter conditions are reversed, in which case, in the absence of force feedback the boom will be bistable, moving to either its upper or lower limit stop when released. As plotted in the "Net Moment" chart, a stable net moment curve will intersect the $Q = 0$ axis with a negative slope, i.e. from upper left to lower right. It would also be desirable for the net moment vs. rotation curve to be relatively linear near $\gamma = 0$, in the region where a force-feedback instrument would normally be operating.

The Problem

Most equations dealing with the bending of objects assume that the shape of the object (beam) does not change appreciably from its un-loaded shape when the load forces are applied. For the beams supporting the floors of a building, for example, this is a reasonable assumption. A leaf spring, bent through something like 180 degrees does not

meet the assumption of constant shape, and designers would typically use a Finite Element Analysis program in order to analyze such a spring. The problem would be solved by using successive approximation.

Lacking an FEA program I tried to see if the analysis could be done using the Microsoft spreadsheet program, Excel. The desired analysis can, indeed, be done by using a series of Excel worksheets into which all the relevant parameters are entered. Then, the effects of changing the various dimensions can be studied as they relate to the resulting period of oscillation, linearity and stability.

The problem of determining the physical characteristics of a system of the STS-1 design may be broken into several parts.

1. Given the dimensions and materials of the boom components and of the seismic mass blocks, compute the effective mass value, the location of the center of mass, the radius of gyration and the rotational moment of inertia for the boom-mass system.

This is done in worksheet "Geometry". These values change whenever the dimensions, geometry or materials of the boom or seismic mass are changed.

2. As a function of the boom rotation angle, γ (0 = horizontal rest position), compute the coordinates of the center of mass and spring attachment point and the angle of the spring attachment. Worksheet "Rotation" handles this. Line 6 contains the results of the computation for a particular boom angle, defined in cell A69 of worksheet “Control” (Control!A69).

3. As a function of the leaf spring dimensions and material properties, compute the spring end force (magnitude and direction) and the spring end moment, given the spring-boom attachment point coordinates and angle.

This step is the most involved to describe. For computational purposes, the leaf spring is broken into 600 small but finite elements of identical length.

Beginning at the point where the spring is attached to the mounting base, the bending moment is calculated for the first element which allows the bend radius, ‘r’, to be determined fot the vicinity of that point. Then knowing the location and angle of the element's starting face, the location and angle of its opposite face may be accurately computed. In

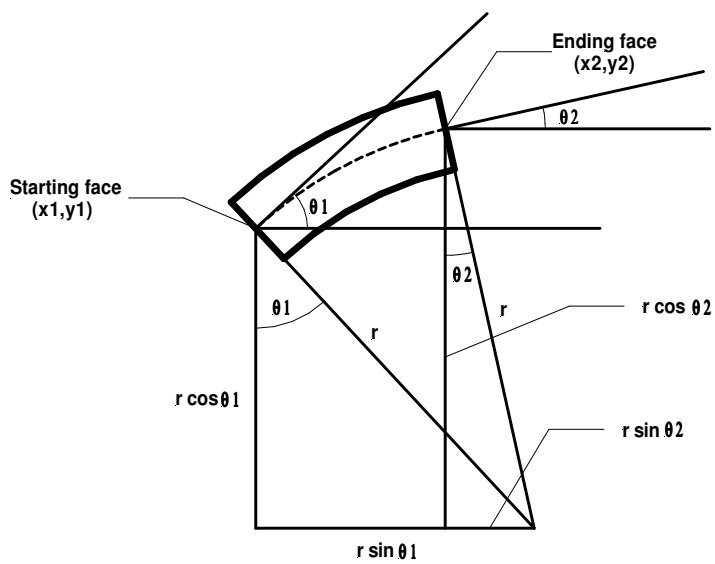


Fig. 5

turn, the ending coordinates and angles of each element are used as the starting conditions for the next. In this way, the shape of the entire spring may be built up, element by element. Worksheet "spring" does this computation, using 600 lines, one for each of the elements, and the resultant spring shape is graphed on the chart "shape".

For convenience in calculation (avoiding circular references) it is assumed that the force and moment are applied to the spring at the (fixed) location of the boom attachment point, which will not necessarily start out coincident with the end of the spring. This can be viewed as if the force and moment are applied at the end of a rigid link connected to the end of the spring and extending to the attachment point on the boom.

However, it should be noted that the problem we have just solved is the inverse of what we need. What we wanted were the values of the force and moment given the desired spring-end position. These are obtained in the following step by using the Excel "Solver" to back-solve the problem, performing a systematic trial and error search to find the exact force and moment which will result in the required spring end location and angle. The effect of this will be to make the end of the spring coincide with its attachment point on the boom, and the imaginary link described above, disappears.

To do this for one boom angle and parameter set, click "Run Solver". The results will appear in the "Control" worksheet, as changes to the spring end force and moment in cells C59-61, and also to any variables deriving from them. These changes will also be reflected in charts “Shape”, “Moment” and “Slope”.

4. For each of 21 boom rotation angles from +5 degrees to -5 degrees, compute the net restoring moment on the boom. This process has been automated with the macro "Spring_Solve", which when invoked by clicking it, first adjusts the mass blocks for exact balance at $\gamma = 0$, and then goes on using the Excel Solver to compute the significant moment values for the 21 values of gamma. The results are placed in the Results Table, from which they become the source data for the three charts "Net Moment", "Pivot Moments" and "Spring Moments" A copy of the important parameters is created adjacent to the Results Table, for convenience in archiving the data set.

5. Plot the net restoring moment vs. boom angle. Then compute the slope of the net restoring moment curve at the point of boom equilibrium.

Worksheet "CurFitter", generates a smoothed version of the net moment around the boom pivot vs. boom position, which is also displayed on charts "Pivot Moments" and "Net Moment" along with the previously computed results.

6. Finally, compute the free period based on the boom moment of inertia and slope of the net restoring moment curve.

This is calculated in worksheet "Control" and uses the slope of the net moment data obtained in step 5 and the moment of inertia from step 1 to obtain the free period of the spring-mass for the selected set of parameters. The period is displayed in cell C38.

Notes on the Excel analysis system

The worksheet "Control" is where most parameters are entered and where the resulting values are viewed. Cells which contain parameters (constants) are yellow. Cells which contain key results (formulas) are blue.

When "Spring Solve" finishes calculations for all 21 values of γ, the results may be saved by clicking "Save Plot Data". This appends the resulting data and parameter set to worksheet "Data Sets". Then the process may be repeated using different parameters.

To retrieve a previously saved data set along with its parameters, go to the "Data Sets" worksheet. Highlight (click on) the blue date/time cell at the beginning of the desired data set and click "Load Plot Data". This will copy the selected data set and associated parameters back to the "Control" worksheet.

Results

The preliminary results of this investigation are quite interesting. Needing a set of dimensions to analyze and not having a spare STS-1 in my scrap box, I scaled various photographs and used some of the dimensions given in "The Paper"[1]. In addition, S-T Morrisey[2] had indicated that the effective mass was around 0.6 Kg and that the free period was set up for about 6 sec. I am still seeking more information on the STS-1 geometry, though apparently what I have assumed so far is not too far off the mark.

Assuming a single steel spring 0.009" thick x 72.9 mm wide and 185.67 mm long, with the other dimensions approximately as given in "The Paper"[1], the resulting period is 6.00 seconds, requiring a 0.597 Kg effective mass. Also of interest is how changes of only a tenth of a millimeter in the spring length can dramatically affect the free period. These data were obtained using the above dimensions and varying the spring length.
We see that the free period is a function of the leaf spring length, and gets longer as the spring gets shorter until we pass the point of neutral stability and the period numbers jump negative. A negative period indicates that the associated configuration is unstable and the value suggests how quickly the boom will tend to move toward the end stop.

Spring Length mm	Free Period Seconds	Wt. of Mass lb.
185.90	4.10	0.5957
185.85	4.36	0.5960
185.80	4.67	0.5963
185.75	5.07	0.5966
185.70	5.60	0.5970
185.67	6.00	0.5971
185.64	6.50	0.5973
185.60	7.44	0.5976
185.55	9.47	0.5979
185.50	15.42	0.5982
185.48	26.52	0.5983
185.46	-27.11	0.5985
185.44	-15.53	0.5986
185.40	-10.14	0.5988
185.35	-7.74	0.5992
185.30	-6.50	0.5995

For each spring length selected, the mass value must be adjusted very slightly to obtain an exact balance when the beam is horizontal. In practice, when adjusting the spring length, one would probably want to be fine tuning the mass *position* to balance the boom; but I had wanted to keep the geometry constant for this data set, so for now am choosing to adjust the mass value instead. (although I now see, this still does move the Center of Mass)

It should be noted that there is nothing particularly special about adjusting the spring length to vary the free period, just as there is nothing special about the spring lengths obtained. I believe that a similar looking set of data could be created by keeping the spring length constant and systematically adjusting some other dimension of the system.

In looking at the sensitivity of the system to changes in its various dimensions, it appears that many or most of the other dimensions may be just as critical as the spring length. A 0.1 mm variation from optimum makes an enormous difference in performance. One is unlikely to get this design working well by trial and error methods, and preliminary modeling will almost certainly have to be an essential part of the design process.

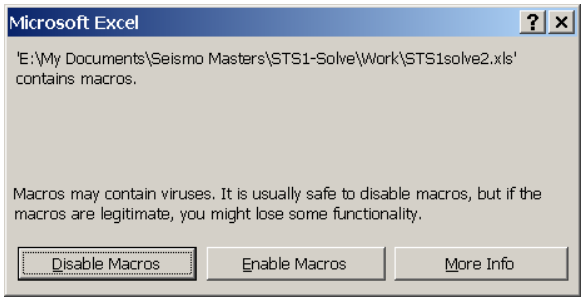
The Excel Cookbook:

Notes:

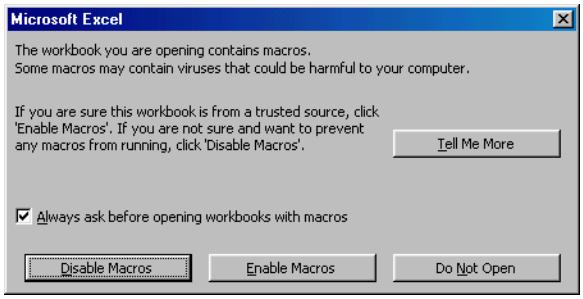
This workbook will only work with Excel 97 and later versions. It is also computationally intensive. The routine "Spring Solve" requires nearly one minute to complete, with a 2 GHz PentiumIV, which implies that "Run Solver" is requiring an average of 2.5 seconds per solution. Excel 2002, when running this workbook, was requiring 20Mb of free memory for the worksheet and the operation of its macros. With a 600 MHz Pentium III, computation was about 3 times slower.

Getting Started:

Macro security:
Since this workbook uses VBA macros you will want to give Excel permission to run them. Some macros contain viruses, though I have tried to make sure that the ones here don't. When opening this workbook, you should agree to enable macros unless you are just browsing.



Newer Version



Older Version

To permit enabling macros, in Excel (newer versions), with a blank worksheet, go to "Tools / Options / Security / Macro Security / Security Level" and select either Low or Medium (recommended), then OK (twice). Note that if you selected Low, Excel will never offer you the option to disable macros for *any* Workbooks you may open.

Some earlier versions of Excel, allow macros all the time. Slightly more recent versions permit you to activate a warning popup (recommended) which allows you to select whether to allow macros to run or not. In those version you can go to "Tools/ Options / General" and checkmark "Macro virus protection". This has exactly the same effect as setting the Security Level to "Medium" as described above.

Solver:
This Workbook requires the Excel Add-in called "Solver" To determine that it will be available, Open Excel with a blank worksheet, then select Tools / Add-ins and look for "Solver Add-in" in the list. It doesn't matter if it is check marked or not, this Worksheet will select it automatically when it is opened.

However, if there is no “Solver Add-in” listed, you can expect that the first time this Workbook is opened, you will be asked to install it. That will require you to have available the original installation files for Excel, either from a CD, your hard drive or from a network.

Custom toolbar:
After opening this Worksheet, look to see that the STS-1 custom toolbar is visible.

If not, select “View / Toolbars ” and make sure the "STS-1" entry is checked.

Worksheet locking.

All worksheets are protected, which prevents the accidental altering of important formulas. Cells which contain input constants are colored light yellow. Data may be entered in those cells, and all macros may be run without unlocking the worksheets. To unlock a worksheet, select Tools/Protection/Unprotect Sheet.

Back up your Worksheet:
Make a backup copy of the STS1solve.xls, or keep STS1solve.zip. This is mainly so that you can get back to a known starting point after doing experiments. Having a backup copy has saved me a lot of work several times, after I'd messed things up.

The Charts:

Shape (Updated real-time. Valid solution after running “Solver”)
This shows the shape of the spring The X and Y coordinate scales should be kept the same to avoid distortion. It is a great place to see how the spring end forces are adjusted to get the desired end coordinates. Whenever you change a parameter, the spring end will move away from the target point (assuming the “Auto Solve” cell was set to “FALSE”). After running Solver, the end forces will have been adjusted so that the spring is connected again.

Net Moment (Plotted from data in the table generated by “Spring Solve”)
Shows the restoring moment vs. boom rotation, plotted from the difference between the gravity moment and spring moment curves, or rather, from cubic polynomial curves fitted to them. The present setup computes the moment data accurately enough for each curve to be essentially identical to its fitted approximation. A line tangent to the Net Moment curve at zero degrees rotation approximates the torsion spring constant, which is determining the free period. Also a plot of slope vs. boom rotation gives a quantitative look at the degree of nonlinearity of this virtual spring.

Pivot Moments (Plotted from data in the table generated by “Spring Solve”)
Shows the gravity and spring moments separately vs. boom rotation. These are opposing moments, but the gravity moment is plotted as its negative. Where the curves intersect is the point where the net moment is zero. At present, “Spring Solve” adjusts the mass slightly, which raises or lowers the gravity moment curve so that, for the assumed set of parameters, the intersection occurs at zero degrees boom rotation. These curves are very helpful in visualizing how geometry changes are affecting the free period.

Spring Moments (Plotted from data in the table generated by “Spring Solve”)
The spring has two effects on the boom. First it creates a force which tends to rotate the boom downward. Then its end moment, which in the default example is about four times larger, tends to rotate the boom upward. The force-induced moment, the spring end moment and their sum are plotted vs. boom rotation. It is interesting that both components of the spring moment, when varying the boom angle, change in opposite directions, so that the total spring moment change with boom rotation is rather small.

Period (Plotted from data in manually-entered table)
shows free period vs. spring length for a set of assumed parameters. It is plotted from the "Control” worksheet, "Summary, current values" data. This data for this chart was entered by hand by running “Spring Solve” for successive values of spring length and copying the results into the table.

Moment (Updated real-time. Valid solution after running “Solver”)
shows the spring bending moment vs. distance along the spring from s = 0 to L, the end of the spring. Data comes from Worksheet "Spring".

Slope (Updated real-time. Valid solution after running “Solver”)
shows the angle θ the spring makes with the world horizontal vs. distance along the spring. Where the spring attaches to the base, θ = 90 degrees, at s = 0, . If the boom is horizontal, θ = -90 degrees, at s = L, where the spring attaches to the boom. Data comes from Worksheet "Spring".

The STS-1 Command Bar:

Auxiliary buttons:

Paste Values
Pastes only copied cell value, not its formula or format.
Almost always you will want to use this instead of <ctrl>V or Edit, Paste.

Paste Format
Pastes only cell format, not its value or formula.
You will likely need this only when modifying the worksheets.

Protect All
Sets all worksheets and charts in this workbook to “Protected”. Normally used before saving the workbook file, if worksheets have been unprotected for editing.

Spring solving buttons:

Run Solver
Recomputes the spring end force and moment to match the current boom position and other parameters. Attaches the spring end to the target position. Run this after changing any parameter. Usually takes 10-30 seconds.

Spring Solve
Computes and records a complete set of 21 results for boom positions from -5 deg to +5 deg. using the current parameter set in Control!C2:C12, etc. The result table is saved in Control!A72:K92 and is used to create the three "Moment" graphs. A copy of the parameters used is placed to the right of the data set. Takes about 10 minutes on a Pentium II-class computer and about one minute on a 2GHz Pentium IV.

Save Plot Data
Appends the current “Plot Data” results table and parameters to Worksheet "Data Sets"

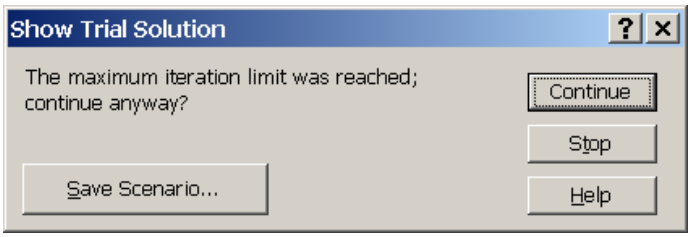
Load Plot Data
Retrieves one value set from the "Data Sets" worksheet into the “Plot Data” table for display in the "Moments" graphs, along with the associated parameters. Loads the table corresponding to the selected (blue) Date/Time cell.

Set Chart Titles
Updates the Chart Title variables to correspond to the data sets being displayed. Use this before printing or using the charts. However it will be done automatically by “Run Solver”, “Spring Solve” and “Load Plot Data”

Auto Solve
Almost any time an input parameter (yellow cell) is changed, the spring end becomes detached from its attachment point on the boom, which can best be observed in the Chart “Shape”. In order to reconnect the spring one must execute the “Run Solver” macro (above), which iteratively seeks the solution. This procedure can be automated by entering “TRUE” in cell H66 of Worksheet “Control”. Then, any change to a cell which causes the spring end to separate from its attachment will automatically invoke “Run Solver”. By default, H66 is set “FALSE”.

Solver Convergence
Although the Excel "Solver" program usually does an excellent job of converging on the correct solution, it is possible for it to become confused if its starting point is badly chosen or following large changes made to the parameters. If necessary, the starting values may be entered manually into cells C59-61 of the "Control" worksheet. These cells normally contain the values calculated in the previous "Solver" run.

To observe the sort of problem which may occur, make sure cell H66 of "Control" is "FALSE"; then try entering 0, 0, -1 for F, ϕ and Q(L). You can see on the "Shape" chart that the spring is now curving off the chart to the left. Then click the "Run Solver" button. After 100 iterations, Solver will pause, and you should click "Stop" in the pop up window which appears.



Now look at "Shape" again.

This sort of problem can be avoided by ensuring that the starting conditions have the spring bending in the correct direction and ending in the general vicinity of the boom attachment point. Normally that will be the case if changes made to the parameters are not too large. However if there is a problem, for the default spring defined here, 0,0,2 can be entered as a safe starting set. Note that entering 0,0,0 creates undefined values which are associated with having specified an infinite bending radius.

Known bugs:

1) If you start to close the worksheet, but click "cancel" on the "Do you want to save changes" pop-up, the special "STS-1" toolbar disappears.

Workaround to recover it: Select Tools / Customize / Toolbars Then scroll down and restore the checkmark for "STS-1".

Some Random Thoughts:

Thin beams: A wide, thin, spring behaves slightly differently when bent compared with a piece of steel that has, for example, a square cross section. The simple beam-bending equations would be accurate for the square beam, but not for the leaf spring. The latter will appear to be stiffer than predicted by a factor of about 9%, or to be more precise by a factor of $1/(1-\nu^2)$ where ν is Poisson's ratio, which for steel is about 0.29. The easiest way to incorporate this correction into the bending equations is to assume an increased value for the modulus of elasticity of the spring material by multiplying by that factor. If

the stated modulus of elasticity for the spring material is E, then the corrected value, called E_1 in the spreadsheet, equals $E/(1-\nu^2)$ or for steel, E/ 0.916 .

Temperature coefficient: At a guess, the temperature coefficient of the mechanical system should be a little less than 200 parts per million per degree C, whatever that implies in performance terms. I think it means that you would have problems with temperature related drift, especially in a VBB system. It appears that the real STS-1 may be using a spring alloy, processed to have a very low tempco of elastic modulus such as Elinvar or NiSpan-C. However, it should be noted that these alloys are not quite as strong as spring steel (lower yield stress) and would be more prone to taking a permanent 'set' if bent too stongly. Constructing, aging, testing and adjusting such a spring system was likely a significant part of the effort of building the STS-1.

An open issue for me is how to connect the force transducers and position sensor to the boom in a way that isn't badly affected by the short radius of their attachment arms as my photos aren't clear as to how it is done in the STS-1 design.

Brett Nordgren
20 May, 2000
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Bibliography:

1 Wielandt, E. & Streckeisen, G., 1982.
The leaf-spring seismometer: design and performance, Bull. Seism. Soc. Am., 72(6), 2349-2367.

2 Morrissey, S.-T., 26 Mar 2000
Correspondence archived on Public Seismic Network
http://psn.quake.net/info/stm-mail.zip
http://www.eas.slu.edu/People/STMorrissey/index.html

Appendix I

The Solver Model

The macros "Run_Solver" and "Spring_Solve" both use the "Solver" add-in program. The control parameters passed to "Solver" are located on the "Control" worksheet in the named range "Solver_Model" located in cells I60-64. The data in those cells is as follows:

Cell	Contents	Meaning
I60	=MIN($F$65)	Minimize F65 (θ(L) error squared)
I61	=COUNT($C$59:$C$61)	By varying cells C59-61
I62	=$D$63=Control!$C$63	Constraint: X(L) = its target value
I63	=$D$64=Control!$C$64	Constraint: Y(L) = its target value
I64	={200,100,0.0000000001,0.00000000001,FALSE,FALSE,TRUE,2,2,1, 0.0000000001,FALSE}	

Cell I64 contains multiple parameters:

200	Max time – sec.
100	Max number of Iterations
0.0000000001	Precision
0.0000000001	Tolerance
FALSE	Assume linear (not)
FALSE	Assume non-negative (not)
TRUE	Use automatic scaling
2	Estimates: Quadratic (selection 2)
2	Derivatives: Central (selection 2)
1	Search: Newton (selection 1)
0.0000000001	Convergence
FALSE	Show iteration results (not)

These values have been found to generally work pretty well. On a slower computer, it may be necessary to reduce "Max Time" and "Iterations" to permit the process to abort if it is taking too long.

Appendix II

To Understand the Math:

It is not necessary to understand the math, physics or mechanics which were used to create the spreadsheet in order to use it, but the more background the reader has in certain areas, the better the problem can be understood.

To begin with, the over all design is analyzed as a torsion pendulum, so it is useful to have seen something of the physics of such a device. In particular, the analysis considers this pendulum to be a distributed mass, rather than a point mass, which involves such concepts as the Radius of Gyration. A beginning physics book would cover these.

Secondly, a basic understanding of forces, force vectors and moments is fundamental to the analysis. It is useful to understand how to interpret a free-body diagram, such as in fig. 3. This involves understanding the relationship between moments and forces and understanding that, when at rest, the forces and moments on such a body must both sum to zero. These would be covered in a beginning book on Mechanics or Strength of Materials.

Also, such books would cover the simple beam-bending equations used to compute the bending radius of each element of the spring, as shown in figure 6, which uses concepts such as material Elastic Modulus, and properties of the spring cross-section shape, such as Section 'Moment of Inertia' (which incidentally has nothing to do with physical inertia).

And algebra and trigonometery are used throughout.